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Local Synthesis and Tooth Contact Analysis of Face-Milled Spiral Bevel Gears

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NOMENCLATURE

[A]	Augmented matrix of linear equation system
A^*	Distance between the shifted center of bearing contact and pitch apex
A_m	Mean cone distance
$a_{ij} \quad (i,j = 1,2,3)$	Coefficients of basic linear equations
a, b	Half-long and short axes of contact ellipse
b_G	Gear dedendum
c	Clearance
C	Coefficient of the second order of Taylor series of generation motion
D	Coefficient of the third order of Taylor series of generation motion
E	Coefficient of the forth order of Taylor series of generation motion
F	Coefficient of the fifth order of Taylor series of generation motion
$6CX$	Third order parameter of generation motion
$24DX$	Forth order parameter of generation motion
$120EX$	Fifth order parameter of generation motion
E_{mi}	Blank offset in generation of gear i
$\vec{e}_f, \vec{e}_h$	Principal directions of surface Σ_1
$\vec{e}_s, \vec{e}_q$	Principal directions of surface Σ_2
$\vec{e}_{sp2}, \vec{e}_{qp2}$	Principal directions of gear surface in system S_{p2}
H_G, V_G	Gear horizontal and vertical settings
h_m	Mean whole tooth depth
i	Tilt angle
j	Swivel angle
$K_\Sigma^{(i)} \quad (i = 1,2)$	Sum of principal curvatures of surface 1 or 2

$K_I^{(i)}, K_{II}^{(i)} \quad (i = 1,2)$	Principal curvatures of surface 1 or 2
$[L_{yx}]$	Matrix of coordinate transformation from system S_x to system S_y for free vectors
$m'_{21}(\phi_1)$	Derivative of transmission ratio
M	Mean contact point
$[M_{yx}]$	Matrix of coordinate transformation from system S_x to system S_y for position vectors
$N_i \quad (i = 1,2)$	Number of teeth of pinion $(i = 1)$ or gear $(i = 2)$
$\vec{n}_{p2}$	Unit normal vector of gear cutter surface in system S_{p2}
$\vec{n}$	Common unit normal at point of contact
O_i	Pitch cone apex of gear i
O_{2R}	Root cone apex of gear
p	Percentage of amount of shift along the pitch line over face width
PW	Point width of gear cutter
q_i	Cradle angle for gear i
R_{cp}	Point radius of pinion head cutter
R_{aG}	Gear ratio of roll
R_{u2}	Gear nominal cutter radius
r_c	Gear cutter tip radius
$\vec{r}_{p2}$	Position vector of gear cutter surface in system S_{p2}
$\vec{r}_i$	Position vector of tooth surface of gear i represented in system S_i, $\vec{r}_i$ is equivalent to $[r_i]$
$\vec{r}_h^{(i)}$	Position vector of mean contact point in system S_h
$\vec{r}_h^{(O_F)}$	Position vector of pinion cutter center in system S_h
$\vec{r}_1(\theta_F, \phi_F)$	Position vector of pinion in system S_1
$\vec{r}_2(\theta_G, \phi_p)$	Position vector of gear in system S_2

S_a, S_b	Coordinate systems originated at point of contact between Σ_1 and Σ_2
S_{mi}	Coordinate system rigidly connected to the cutting machine of gear i
S_{ci}	Movable coordinate system rigidly connected to the cradle of cutting machine for gear i
S_h	Fixed coordinate system
S_i	Coordinate system rigidly connected to gear i
(s_G, θ_G)	Surface coordinates of gear cutter surface
(s_F, θ_F)	Surface coordinates of pinion cutter surface
S_{ri}	Radial setting of gear i
S_x	Auxiliary coordinate system identified by subscript x
$\triangle T$	Cam setting
$\vec{V}_r^{(i)} \quad (i = 1,2)$	Sliding velocity of contact point in the motion over surface Σ_i
$\vec{V}_{tr}^{(i)} \quad (i = 1,2)$	Transfer velocity of contact point in the motion with surface Σ_i
$\vec{v}_s^{(i)}, \vec{v}_q^{(i)} \quad (i = 1,2)$	Projection of $\vec{V}_r^{(i)}$ upon $\vec{e}_s$ and $\vec{e}_q$
$\vec{V}_{12}$	Relative velocity at contact point
$\vec{v}_{m2}^{(p2)}$	Relative velocity in the process for gear generation represented in system S_{m2}
$(X_o^{(i)}, Z_o^{(i)})$	Coordinates of center of the arc blade
X_{Bi}	Sliding base for generation of gear i
X_{Gi}	Machine center to back for generation of gear i
(XL, RL)	Parameters determining mean contact point
V_G, H_G	Vertical and horizontal adjustments for the gear drive
Z_R	Distance of gear root cone apex beyond pitch cone apex
κ_s, κ_q	Principal curvatures of surface Σ_2
$\kappa_s^{(p)}, \kappa_q^{(p)}$	Principal curvatures of surface Σ_2
κ_f, κ_h	Principal curvatures of surface Σ_1
γ_{mi}	Machine root angle for generation of gear i

Γ_i	Pitch angle of gear i
γ_i	Root angle of gear i
α	Cam guide angle
α_G, α_F	Cutter blade angles for gear and pinion respectively
ρ	Radius of circular arc
(λ, θ_F)	Surface coordinates of the surface of revolution generated by circular arc blade
η_i	Direction angle of contact path on surface Σ_i
$(\vec{\eta}, \vec{\zeta})$	Unit vectors along long and short axes of contact ellipse
δ	Elastic approach
δ_G	Gear dedendum angle
ϕ_i	Angle of rotation of gear i in the process for generation
ϕ_i'	Rotation angle in meshing of gear i between the gear (2) and the pinion (1)
ϕ_F, ϕ_p	Rotation angles of cradle in the process for pinion and gear generation, respectively
ψ_G	Gear spiral angle
(θ_G^*, ϕ_p^*)	Surface coordinates of gear tooth surface at mean contact point
$\sigma^{(12)}$	Angle formed between principal directions $\vec{e}_f$ and $\vec{e}_s$ (in meshing and generation)
Σ_i	Surface of gear i
Σ_F	Pinion generating surface
Σ_p	Gear generating surface
$\vec{\omega}^{(i)} \quad (i = 1,2)$	Angular velocity of surface Σ_i (in meshing and generation)
$\vec{\omega}^{(F)}, \vec{\omega}^{(P)}$	Angular velocity of the cradle in the process for pinion and gear generation, respectively
$\vec{\omega}_{m2}^{(P2)}$	Relative angular velocity in the process for gear generation represented

	in system S_{m2}
$\omega^{(F1)}$	Relative angular velocity in the process of pinion generation
$\vec{\omega}^{(i)}$	Angular velocity of gear i
$\vec{\omega}^{(ij)}$	Relative angular velocity between gear i and gear j

SUMMARY

Computerized simulation of meshing and bearing contact for spiral bevel gears and hypoid gears [1,2] is a significant achievement that could improve substantially the technology and the quality of the gears. This report covers a new approach to the synthesis of face-milled spiral bevel gears and their tooth contact analysis. The proposed approach is based on the following ideas proposed in [3] (i) application of the principle of local synthesis that provides optimal conditions of meshing and contact at the mean contact point M and in the neighborhood of M; (ii) application of relations between principle directions and curvatures for surfaces being in line contact or in point contact.

The developed local synthesis of gears provides (i) the required gear ratio at M; (ii) a localized bearing contact with the desired direction of the tangent to the contact path on gear tooth surface and the desired length of the major axis of contact ellipse at M; (iii) a predesigned parabolic function of a controlled level (8-10 arc seconds) for transmission errors; such a function of transmission errors enables to absorb linear functions of transmission errors caused by misalignment [3] and reduce the level of vibrations.

The proposed approach does not require either the tilt of the head-cutter for the process of generation or modified roll for the pinion generation. Improved conditions of meshing and contact of the gears can be achieved without the above mentioned parameters. The report is complemented with a computer program for determination of basic machine-tool settings and tooth contact analysi· for the designed gears. The approach is illustrated with a numerical example.

The contents of the following sections cover the following topics:

(1). Basic ideas of local synthesis of gears and the mathematical concept of this approach (Chapter 1). The local synthesis discussed in this chapter is applicable for all types of gears and provides the optimal conditions of meshing and contact at the mean point of tangency of gear tooth surfaces.

(2). Methods for generation of the pinion and the gear and basic machine-tool settings that are

necessary for gear generation (Chapter 2).

(3). Determination of geometry of gear tooth surface, the gear mean contact point and the principal directions and curvatures at this point (Chapter 3).

(4). Application of basic principles of local synthesis for spiral bevel gears (Chapter 4).

(5). Determination of pinion machine-tool settings considering as given:(i) the gear geometry, and (ii) the conditions of meshing and contact at the mean contact point obtained from the local synthesis (Chapter 5).

(6). Computerized simulation of meshing and contact (Tooth Contact Analysis) for spiral bevel gears that have been synthesized in the previous chapters (Chapter 6).

(7). Analysis of the shift of bearing contact caused by the misalignment of gears (Chapter 7).

(8). The theory of modified roll (variation of cutting ratio in the process for generation) and mechanisms used for application of modified roll (Appendix A).

(9). Description of developed computer programs and numerical examples that illustrates the application of those programs.

1 Local Synthesis of Gears (General Concept)

1.1 Introduction

The main goals of local synthesis are to provide: (i) contact of gear tooth surfaces at the mean point of contact of gear tooth surfaces, and (ii) improved conditions of meshing within the neighborhood of the mean contact point. The local synthesis is the first stage of the global synthesis with a goal to provide improved conditions of meshing for the entire area of meshing. The criteria of conditions of meshing are the transmission errors and the bearing contact. The principles of local synthesis that are discussed in this chapter for face-milled spiral bevel gears can be applied for other types of gears as well.

1.2 Basic Linear Equations

Consider two right-handed trihedrons $S_a(\vec{e}_f, \vec{e}_h, \vec{n})$ and $S_b(\vec{e}_s, \vec{e}_q, \vec{n})$ (Fig. 1.2.1). The *common* origin of the trihedrons coincides with the contact point M, the n-axis represents the direction of the surface unit normal, $\vec{e}_f$ and $\vec{e}_h$ are the unit vectors of the principal directions of surface Σ_1, $\vec{e}_s$ and $\vec{e}_q$ represent the principal directions of surface Σ_2, and $\sigma^{(12)}$ is the angle formed between $\vec{e}_f$ and $\vec{e}_s$ (measured clockwise from $\vec{e}_s$ to $\vec{e}_f$ and counterclockwise from $\vec{e}_f$ to $\vec{e}_s$). In reference [4] three linear equations were derived that relate the velocity $\vec{v}_r^{(1)}$ of the contact point over surface Σ_1 with the principal curvatures and directions of contacting surfaces and the transfer components of velocities. These equations are:

$$\left.\begin{aligned} a_{11}v_s^{(1)} + a_{12}v_q^{(1)} &= a_{13} \\ a_{12}v_s^{(1)} + a_{22}v_q^{(1)} &= a_{23} \\ a_{13}v_s^{(1)} + a_{23}v_q^{(1)} &= a_{33} \end{aligned}\right\} \qquad (1.2.1)$$

Here (see the designations in [4])

$$
\begin{aligned}
a_{11} &= \kappa_s - \kappa_f \cos^2 \sigma^{(12)} - \kappa_h \sin^2 \sigma^{(12)} \\
a_{12} &= a_{21} = \frac{\kappa_f - \kappa_h}{2} \sin 2\sigma^{(12)} \\
a_{13} &= a_{31} = -\kappa_s v_s^{(12)} - \left[\vec{\omega}^{(12)} \vec{n} \vec{e}_s\right] \\
a_{22} &= \kappa_q - \kappa_f \sin^2 \sigma^{(12)} - \kappa_h \cos^2 \sigma^{(12)} \\
a_{23} &= a_{32} = -\kappa_q v_q^{(12)} - \left[\vec{\omega}^{(12)} \vec{n} \vec{e}_q\right] \\
a_{33} &= \kappa_s \left(v_s^{(12)}\right)^2 + \kappa_q \left(v_q^{(12)}\right)^2 - \left[\vec{n}\ \vec{\omega}^{(12)}\ \vec{v}^{(12)}\right] - \vec{n}\left[\left(\vec{\omega}^{(1)} \times \vec{v}_{tr}^{(2)}\right) - \left(\vec{\omega}^{(2)} \times \vec{v}_{tr}^{(1)}\right)\right] \\
&+ \left\{\left[\left(\omega^{(1)}\right)^2 m_{21}'(\vec{n} \times \vec{k}_2)\right] \cdot (\vec{r} - \vec{R})\right\} \\
v_s^{(1)} &= \vec{v}_r^{(1)} \cdot \vec{e}_s \\
v_q^{(1)} &= \vec{v}_r^{(1)} \cdot \vec{e}_q
\end{aligned}
\tag{1.2.2}
$$

Equations (1.2.1) and (1.2.2) can be applied for two cases where: (i) surfaces Σ_1 and Σ_2 are in line contact, and (ii) the surfaces are in point contact. The instantaneous line of contact is typical for the case when the gear tooth surface (Σ_1) is generated by the tool surface (Σ_2). The instantaneous point of contact is typical for gears with localized bearing contact.

Line Contact

When the gear tooth surfaces are in line contact, the direction of velocity $\vec{v}_r^{(1)}$ can be varied, and equations (1.2.1) can not provide a unique solution for the unknowns $v_s^{(1)}$ and $v_q^{(1)}$. This results in that the rank of the augmented matrix

$$
[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}
\tag{1.2.3}
$$

must be less than 2. This requirement yields

$$
\left.
\begin{aligned}
a_{12}^2 &= a_{11}a_{22} \\
a_{11}a_{23} &= a_{12}a_{13} \\
a_{12}a_{33} &= a_{13}a_{23}
\end{aligned}
\right\} \qquad (1.2.4)
$$

Equivalent equations are

$$
\left.
\begin{aligned}
a_{11} &= \frac{a_{13}^2}{a_{33}} \\
a_{12} &= \frac{a_{13}a_{23}}{a_{33}} \\
a_{22} &= \frac{a_{23}^2}{a_{33}}
\end{aligned}
\right\} \qquad (1.2.5)
$$

Using equations (1.2.5) and (1.2.2) we obtain equations that will enable us to determine $\sigma^{(12)}$, κ_f and κ_h for Σ_1 considering as given κ_s and κ_q for surface Σ_2. The equations are:

$$
\tan 2\sigma^{(12)} = \frac{2a_{13}a_{23}}{a_{23}^2 - a_{13}^2 + (\kappa_s - \kappa_q)a_{33}} \qquad (1.2.6)
$$

$$
\kappa_f - \kappa_h = \frac{2a_{13}a_{23}}{a_{33}\sin 2\sigma^{(12)}} \qquad (1.2.7)
$$

$$
\kappa_f + \kappa_h = (\kappa_s + \kappa_q) - \frac{a_{13}^2 + a_{23}^2}{a_{33}} \qquad (1.2.8)
$$

Equation (1.2.6) provides two solutions : $\sigma_1^{(12)}$ and $\sigma_2^{(12)} = \sigma_1^{(12)} + \pi/2$ and both of them can be used for computations of κ_f and κ_h that are represented by equations (1.2.7) and (1.2.8). Fig.1.2.2 shows the orientation of two couples of unit vectors $\vec{e}_f^{(i)}, \vec{e}_h^{(i)}$ $(i = 1, 2)$, with respect to unit vector $\vec{e}_s$. The magnitude of principal curvature for the direction with collinear vectors $\vec{e}_f^{(1)}$ and $\vec{e}_h^{(2)}$ is the

same ($\kappa_f^{(1)} = \kappa_h^{(2)}$) although the notation for the unit vectors has been changed. Similarly, we can say that $\kappa_h^{(1)} = \kappa_f^{(2)}$.

Knowing the angle $\sigma^{(12)}$, and the unit vectors $\vec{e}_s$ and $\vec{e}_q$, the principal directions on surface Σ_1 can be determined with the following equations,

$$\vec{e}_I^{(1)} \equiv \vec{e}_f = \cos\sigma^{(12)}\vec{e}_s - \sin\sigma^{(12)}\vec{e}_q \tag{1.2.9}$$

$$\vec{e}_{II}^{(1)} \equiv \vec{e}_h = \sin\sigma^{(12)}\vec{e}_s + \cos\sigma^{(12)}\vec{e}_q \tag{1.2.10}$$

Point Contact

In the case of instantaneous point of contact, the direction of motion of the contact point over the surface is definite, equations (1.2.2) for the unknowns can provide a unique solution for the unknowns $v_s^{(1)}$ and $v_q^{(1)}$ and the rank of matrix [A] is 2. This yields that

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0 \tag{1.2.11}$$

Equation (1.2.11) yields the following relation

$$f(\kappa_s, \kappa_q, \kappa_f, \kappa_h, \sigma^{(12)}, m'_{12}) = 0 \tag{1.2.12}$$

Our goal is to determine κ_f, κ_h and $\sigma^{(12)}$ (the principal curvatures and directions of Σ_1) and provide at the mean contact point (i) a certain direction of the tangent to contact path on surface

Σ_2 , (ii) a desired length of the major axis of instantaneous contact ellipse, and (iii) a parabolic function of transmission errors. For these purpose we have to derive extra equations in addition to equation (1.2.12)

Determination of m'_{21}

The derivative $m'_{21}(\phi_1)$ is the second derivative of function $\phi_2(\phi_1)$ that is taken at the mean contact point; ϕ_1 and ϕ_2 are the angles of rotation of gears 1 and 2. In the case of an ideal gear train, function $\phi_2(\phi_1)$ is linear and is represented by

$$\phi_2 = \phi_1 \frac{N_1}{N_2} \tag{1.2.13}$$

However, due to misalignment between the meshing gears the real function $\phi_2(\phi_1)$ becomes a piecewise periodic function with the period equal to the cycle of meshing of a pair of teeth (Fig. 1.2.3). Due to the jump of angular velocity at the junction of cycles, the acceleration approaches to an infinitely large value and this can cause large vibration and noise. For this reason it is necessary to predesign a parabolic function of transmission error that can absorb a linear function of transmission error and reduce the jump of angular velocity and acceleration [3]. This goal(the predesign of a parabolic function) can be achieved with certain relations between the principal curvatures of contacting surfaces .

Fig. 1.2.4 shows the predesigned transmission function for the gear convex side (Fig. 1.2.4(a)) and gear concave side (Fig. 1.2.3(b)). Both functions-$\phi_2(\phi_1)$ and $\phi_2^{(t)}(\phi_1)$- are in tangency at the mean contact point and have the same derivative m_{21} , at this point.

Consider now that the predesigned transmission function is represented as

$$\phi_2 - \phi_2^{(0)} = F(\phi_1 - \phi_1^{(0)}) \tag{1.2.14}$$

Here: $\phi_1^{(0)}$ and $\phi_2^{(0)}$ are the initial angles of rotation of gears 1 and 2 that provide the tangency of gear tooth surfaces at the mean contact point M.

Using the Taylor expansion up to the members of second order, we obtain

$$
\begin{aligned}
F(\phi_1 - \phi_1^{(0)}) &= \frac{\partial F}{\partial \phi_1}(\phi_1 - \phi_1^{(0)}) + \frac{1}{2}\frac{\partial^2 F}{\partial \phi_1^2}(\phi_1 - \phi_1^{(0)}) \\
&= m_{21}(\phi_1 - \phi_1^{(0)}) + \frac{1}{2} m'_{21}(\phi_1 - \phi_1^{(0)})^2
\end{aligned}
\qquad (1.2.15)
$$

where $m_{21}(\phi_1)$ is equal to N_1/N_2 at the mean contact point and m'_{21} is the to be chosen constant value: positive for the gear concave side, and negative for the gear convex side. The synthesized gears rotates with a parabolic function of transmission errors represented by

$$\triangle\phi_2(\phi_1) = \frac{1}{2} m'_{21}(\phi_1 - \phi_1^{(0)})^2 \qquad (1.2.16)$$

where

$$-\frac{\pi}{N_1} \le (\phi_1 - \phi_1^{(0)}) \le \frac{\pi}{N_1}$$

Equation (1.2.16) enables the determination of m'_{21} considering as known the expected values of transmission errors.

<u>Relation between Directions of Paths of Contact</u>

We recall that velocities $\vec{v}_r^{(1)}$ and $\vec{v}_r^{(2)}$ are related by the equation [4],

$$\vec{v}_r^{(2)} = \vec{v}_r^{(1)} + \vec{v}^{(12)} \qquad (1.2.17)$$

Directions of velocities $\vec{v}_r^{(1)}$ and $\vec{v}_r^{(2)}$ coincide with the tangents to the contact path that form angles η_1 and η_2 with the unit vector $\vec{e}_s$ (Fig. 1.2.5). Equations (1.2.17) yield

$$v_s^{(2)} = v_s^{(1)} + v_s^{(12)} \qquad v_q^{(2)} = v_q^{(1)} + v_q^{(12)} \tag{1.2.18}$$

According to Fig. 1.2.5

$$v_q^{(i)} = v_s^{(i)} \tan \eta_i \tag{1.2.19}$$

Third equation of system (1.2.1) and equations (1.2.18) and (1.2.19) yield

$$\tan \eta_1 = \frac{-a_{31} v_q^{(12)} + (a_{33} + a_{31} v_s^{(12)}) \tan \eta_2}{a_{33} + a_{32}(v_q^{(12)} - v_s^{(12)} \tan \eta_2)} \tag{1.2.20}$$

$$v_s^{(1)} = \frac{a_{33}}{a_{13} + a_{23} \tan \eta_1} \tag{1.2.21}$$

$$v_q^{(1)} = \frac{a_{33} \tan \eta_1}{a_{13} + a_{23} \tan \eta_1} \tag{1.2.22}$$

Prescribing a certain value for η_2 (choosing the direction for path of contact on Σ_2), we can determine $\tan \eta_1$, $v_s^{(1)}$ and $v_q^{(1)}$. We recall that coefficients a_{31}, a_{32} and a_{33} do not depend on the to-be determined principal curvatures κ_f and κ_h and $\sigma^{(12)}$.

Relations between the Magnitude of Major Axis of Contact Ellipse, Its Orientation and Principal Curvatures and Directions of Contacting Surfaces

Our goal is to relate parameters $\sigma^{(12)}$,κ_f and κ_h of the pinion surface Σ_1 with the length of the major axis of the instantaneous contact ellipse. This ellipse is considered at the mean contact point and the elastic approach δ of contacting surfaces is considered as known from the experimental data. The derivation of the above mentioned relations is based on the following procedure

Step 1: Using equations (1.2.2), we obtain

$$
\left.\begin{aligned}
a_{11} + a_{22} &= K_\Sigma^{(2)} - K_\Sigma^{(1)} \equiv K_\Sigma \\
a_{11} - a_{22} &= g_2 - g_1 \cos 2\sigma^{(12)} \\
(a_{11} - a_{22})^2 + 4a_{12}^2 &= g_2^2 - 2g_1 g_2 \cos 2\sigma^{(12)} + g_1^2
\end{aligned}\right\} \tag{1.2.23}
$$

Step 2: It is known from [4] that

$$
a = \sqrt{\left|\frac{\delta}{A}\right|} \tag{1.2.24}
$$

$$
A = \frac{1}{4}\left[K_\Sigma^{(1)} - K_\Sigma^{(2)} - \sqrt{g_1^2 - 2g_1 g_2 \cos 2\sigma + g_2^2}\,\right] \tag{1.2.25}
$$

Equation (1.2.25) yields

$$
[(a_{11} + a_{22} + 4A]^2 = (a_{11} - a_{22})^2 + 4a_{12}^2 \tag{1.2.26}
$$

Step 3: We may consider now a system of three linear equations in unknowns a_{11}, a_{12} and a_{22}

$$
\left.\begin{aligned}
v_s^{(1)} a_{11} + v_q^{(1)} a_{12} &= a_{13} \\
v_s^{(1)} a_{11} + v_q^{(1)} a_{22} &= a_{23} \\
a_{11} + a_{22} &= K_\Sigma
\end{aligned}\right\} \tag{1.2.27}
$$

Step 4: The solution of equation system (1.2.27) for the unknowns a_{11}, a_{12} and a_{22} allows to express these unknowns in terms of $a_{13}, a_{23}, K_\Sigma, v_s^{(1)}$ and $v_q^{(1)}$. Then, using equation (1.2.25) we can get the following equation for K_Σ

$$K_\Sigma = \frac{4A^2 - (n_1^2 + n_2^2)}{2A - (n_1 \cos 2\eta_1 + n_2 \sin 2\eta_1)} \tag{1.2.28}$$

Here:

$$n_1 = \frac{a_{13}^2 - a_{23}^2 \tan^2 \eta_1}{(1 + \tan^2 \eta_1) a_{33}}$$

$$n_2 = \frac{a_{13} \tan \eta_1 + a_{23})(a_{13} + a_{23} \tan \eta_1)}{(1 + \tan^2 \eta_1) a_{33}}$$

$$A = \frac{a^2}{\delta} \tag{1.2.29}$$

The advantage of equation (1.2.28) is that we are able to determine K_Σ knowing the major axis $2a$ of the contact ellipse and the elastic approach δ.

Step 5: The sought for principal curvatures and directions for the pinion identified with κ_f, κ_h and $\sigma^{(12)}$ can be determined from the following equations

$$K_\Sigma^{(1)} = K_\Sigma^{(2)} - K_\Sigma \tag{1.2.30}$$

$$\tan 2\sigma^{(12)} = \frac{2a_{22}}{g_2 - (a_{11} - a_{22})} = \frac{2n_2 - K_\Sigma \sin 2\eta_1}{g_2 - 2n_1 + K_\Sigma \cos 2\eta_1} \tag{1.2.31}$$

$$g_1 = \frac{2a_{12}}{\sin 2\sigma^{(12)}} = \frac{2n_2 - K_\Sigma \sin 2\eta_1}{\sin 2\sigma^{(12)}} \tag{1.2.32}$$

$$\kappa_f \equiv \kappa_I^{(1)} = \frac{\kappa_\Sigma^{(1)} + g_1}{2} \tag{1.2.33}$$

$$\kappa_h \equiv \kappa_{II}^{(1)} = \frac{K_\Sigma^{(1)} - g_1}{2} \tag{1.2.34}$$

Step 6: The orientation of unit vector $\vec{e}_f$ and $\vec{e}_h$ is represented with equations (1.2.9) and (1.2.10). The orientation of the contact ellipse with respect to $\vec{e}_f$ is determined with angle $\alpha^{(1)}$ (Fig. 1.2.6) that is represented with the equations

$$\cos 2\alpha^{(1)} = \frac{g_1 - g_2 \cos 2\sigma^{(12)}}{(g_1^2 - 2g_1 g_2 \cos 2\sigma^{(12)} + g_2^2)^{\frac{1}{2}}} \tag{1.2.35}$$

$$\sin 2\alpha^{(1)} = \frac{g_2 \sin 2\sigma^{(12)}}{(g_1^2 - 2g_1 g_2 \cos 2\sigma^{(12)} + g_2^2)^{\frac{1}{2}}} \tag{1.2.36}$$

The minor axis of the contact $(2b)$ ellipse is determined with the equations

$$b = \sqrt{\left|\frac{\delta}{B}\right|} \tag{1.2.37}$$

$$B = \frac{1}{4}\left[K_\Sigma^{(1)} - K_\Sigma^{(2)} + \sqrt{g_1^2 - 2g_1 g_2 \cos 2\sigma + g_2^2}\,\right] \tag{1.2.38}$$

Local Synthesis Computational Procedure :

The following is an overview of the computational procedure that is to-be used for the local synthesis.

The input data are: $\kappa_s, \kappa_q, \vec{e}_s, \vec{e}_q, \vec{r}^{(M)}, \vec{\omega}^{(12)}, \vec{v}^{(12)}$ and δ. The to-be chosen parameters are: η_2, m'_{21} and $2a$. The output data are: $\kappa_f, \kappa_h, \sigma^{(12)}, \vec{e}_f$ and $\vec{e}_h$.

Step 1: Choose η_2 and determine η_1 from equation (1.2.20)

Step 2: Determine $v_s^{(1)}$ and $v_q^{(1)}$ from equations (1.2.21) and (1.2.22)

Step 3: Determine A from equation (1.2.29)

Step 4: Determine K_Σ from equation (1.2.28)

Step 5: Determine $\sigma^{(12)}, \kappa_f$ and κ_h by using the set of equations from (1.2.30) to (1.2.34)

Step 6: Determine the orientation of the contact ellipse and its minor axis by using equations from (1.2.35) to (1.2.37)

1.3 Conclusion

The contact of tooth surfaces is considered for two cases: line contact and point contact. For line contact, the principal directions and curvatures of one surface can be determined in terms of the other's knowing the relative motion between the two . For point contact, we proposed an approach for local synthesis of spiral bevel gears which enables: (i) to provide a limited level of transmission errors, (ii) optimal direction for the path of contact on gear surface Σ_2,and (iii) the guaranteed length of the major axis of contact ellipse.

The output data obtained from the procedure of local synthesis are: $\kappa_f, \kappa_h, \sigma^{(12)}, \vec{e}_f$ and $\vec{e}_h$. The machine-tool settings for the generation of the gear tooth surfaces must be carefully chosen to guarantee the above mentioned conditions of local meshing and contact.

2 Pinion and Gear Generation

2.1 Pinion Generation

To describe the pinion generation we will use the following coordinate system (Fig.2.1.1): (i) S_{m1}- a fixed coordinate system that is rigidly connected to the cutting machine; (ii) S_{c1} -a movable coordinate system that is rigidly connected to the cradle and performs rotation with the cradle about the Z_{m1}- axis; initially, S_{c1} coincides with S_{m1} (Fig.2.1.1 (b)); angle ϕ_F determines the current position of S_{c1} (Fig 2.1.1 (c)) : (iii) Coordinate systems S_a and S_b that are rigidly connected to the cradle and its coordinate system S_{c1}; systems S_a and S_b are used to describe the installment of the head-cutter on the cradle. Angle q_1 determines the orientation of S_a with respect to S_{c1}; (iv) Coordinate system S_F that is rigidly connected to the head-cutter (not shown in Fig.2.1.1); the head-cutter in the process for generation performs rotation with the cradle (transfer motion) and relative motion with respect to the cradle about an axis that passes through O_a; (v) Auxiliary coordinate systems S_d and S_p are used to describe the installment of the pinion on the cutting machine (Fig.2.1.1 and Fig.2.1.2); the pinion axis forms angle γ_{m1} with axis X_d that is parallel to X_{m1}. (vi) A movable coordinate system S_1 that is rigidly connected to the being generated pinion; the pinion rotates about the axis X_p and ϕ_1 is the current angle of pinion rotation (Fig.2.1.2).

Henceforth, we have to differentiate the parameter of motions that are performed in the process for generation and the parameters of installment of the head cutter and the pinion on the cutting machine.

In the process for generation the cradle of the cutting machine with the mounted head-cutter performs rotation with angular velocity $\vec{\omega}^{(F)}$ (Fig.2.1.2). The head-cutter performs rotational motion with respect to the cradle but this motion is not related with the process for generation and just provides the desired velocity of cutting. The being generated pinion performs rotational motion with angular velocity $\vec{\omega}^{(1)}$ (Fig.2.1.2) that is related with $\vec{\omega}^{(F)}$.

The parameters of installment of the head-cutter are: (i) the swivel angle j (Fig.2.1.1) and the

tilt angle i that is the turn angle of S_t about Y_b (Fig.2.1.3); $S_{r1} = |\overline{O_c O_{m1}}|$ is the radial setting; q_1 is the cradle angle.

The parameters of installment of the pinion are: E_{m1}-the shortest machine center distance (Fig. 2.1.1, Fig.2.1.2); root angle γ_{m1}; sliding base X_{B1}; machine center to back X_{G1}.

2.2 Gear Generation

While describing the gear generation, we will consider the following coordinate systems: (i) S_{m2} that is rigidly connected to the cutting machine; (ii) S_{c2} that is rigidly connect to the cradle, (iii) S_{p2} that is rigidly connected to the head-cutter and S_{c2}, (iv) S_{d2} that is an additional fixed coordinate system rigidly connected to S_{m2} ; and (v) S_2 that is rigidly connected to the being generated gear.

The cradle performs rotation about the Z_{m2} axis with angular velocity $\vec{\omega}^{(p)}$ (Fig.2.2.1). The initial and current positions of coordinate systems S_{c2} and S_{p2} with respect to S_{m2} are shown in Fig.2.2.1 (a) and Fig.2.2.1 (b), respectively.

Coordinate system S_{d2} (it is rigidly connected to S_{m2}) is used to describe the installment of the gear at the cutting machine (Fig.2.2.2(a)). In the general case apices O_{2R} and O_2 of the gear root cone and pitch cone do not coincide. Apex O_{2R} is located on axis X_{m2} of the cutting machine. The origin O_{d2} of S_{d2} coincides with the apex O_2 of the gear pitch core. Axes X_{d2} and X_{m2} form angle γ_{m2} which is the gear machine root angle.

Coordinate system S_2 is rigidly connected to the gear that in the process of generation performs rotation about X_{d2} with angular velocity $\vec{\omega}^{(2)}$ (Fig.2.2.2(b)). Angle ϕ_2 is the current angle of rotation of gear 2.

2.3 Gear Machine Tool Settings

Gear Cutting Ratio

Fig.2.3.1 shows the sketch of the gear with noncoinciding apexes of the root and pitch cones. In the process for generation the pitch line $O_2 P$ is the instantaneous axis of rotation. It is evident

that the angular velocity of rotation in relative motion, $\vec{\omega}^{(p2)}$, must lie in the plane that is formed by vectors $\vec{\omega}^{(p)}$ and $\vec{\omega}^{(2)}$ (Fig.2.3.2)

$$\vec{\omega}^{(p2)} = \vec{\omega}^{(p)} - \vec{\omega}^{(2)} \tag{2.3.1}$$

The cutting gear ratio is:

$$R_{aG} = \frac{|\vec{\omega}^{(2)}|}{|\vec{\omega}^{(p)}|} = \frac{\cos\delta_G}{\sin\Gamma_2} = \frac{\cos(\Gamma_2 - \gamma_2)}{\sin\Gamma_2} \tag{2.3.2}$$

Gear Settings

Fig.2.3.3 shows the installment of the head- cutter. We designate the mean pitch cone distance O_2P (Fig.2.3.1, Fig.2.3.3) by A_m. Then we obtain (Fig.2.3.3)

$$H_G = A_m \cos\delta_G - R_{u2}\sin\psi_G \tag{2.3.3}$$

$$V_G = R_{u2}\cos\psi_G \tag{2.3.4}$$

$$S_{r2} = (H_G^2 + V_G^2)^{\frac{1}{2}} \qquad (S_{r2} = |\overline{O_{m2}O_{p2}}|) \tag{2.3.5}$$

$$q_2 = \sin^{-1}\frac{V_G}{S_{r2}} \tag{2.3.6}$$

Here: ψ_G is the spiral angle on the root cone, R_{u2} is the mean radius of the head cutter. The sliding base $|\overline{O_{m2}O_2}|$ is

$$X_{B2} = Z_R \sin\gamma_{m2} \tag{2.3.7}$$

Here: γ_{m2} is the same as the gear root cone angle γ_2. and Z_R is the distance between O_{2R} and O_2, which are the apexes of the root cone and the pitch cone, respectively.

3 Gear Geometry

3.1 Gear Surface

The gear tooth surface is the envelope to the family of generating surfaces. We recall that the cradle carries the head-cutter that is provided with finishing blades. The blades are rotated about the axis of the head-cutter and generate two cone surfaces. Fig.3.1.1 shows one of the cones.

The family of a generating surface (the cone surface) is generated in S_2 while the cradle and being generated gear perform related rotations, about the Z_{m2}-axis and X_2- axis (Fig.2.2.2).

The derivation of the gear tooth surface is based on the following procedure:

Step 1: We represent the cone surface and its unit normal in system S_{p2} (Fig.3.1.1) as follows

$$\vec{r}_{p2} = \begin{bmatrix} (r_c - s_G \sin\alpha_G)\cos\theta_G \\ (r_c - s_G \sin\alpha_G)\sin\theta_G \\ -s_G \cos\alpha_G \\ 1 \end{bmatrix} \tag{3.1.1}$$

$$\vec{n}_{p2} = \frac{\vec{N}_{p2}}{|\vec{N}_{p2}|} \quad ; \quad \vec{N}_{p2} = \frac{\partial \vec{r}_{p2}}{\partial \theta_G} \times \frac{\vec{r}_{p2}}{\partial s_G} \tag{3.1.2}$$

i.e.,

$$\vec{n}_{p2} = \begin{bmatrix} -\cos\alpha_G \cos\theta_G \\ -\cos\alpha_G \sin\theta_G \\ \sin\alpha_G \end{bmatrix} \tag{3.1.3}$$

Here: s_G and θ_G are the surface coordinates; α_G is the blade angle; r_c is the radius of the head-cutter that is measured at the bottom of the blades. It is evident (Fig.3.1.2) that

$$r_c = R_{u2} \pm \frac{PW}{2} \tag{3.1.4}$$

Here: R_{u2} is the nominal radius, PW is the so called point width; the positive sign in (3.1.4) corresponds to the gear concave side and the negative sign corresponds to the gear convex side.

Equations (3.1.1) and (3.1.3) represent both generating cones with $\alpha_G > 0$ for the gear convex side and $\alpha_G < 0$ for the gear concave side.

Step 2: The family of generating surfaces that is generated in S_2 is represented by the following matrix equation

$$\vec{r}_2(s_G, \theta_G, \phi_p) = [M_{2d_2}][M_{d_2m_2}][M_{m_2c_2}][M_{c_2p_2}]\vec{r}_{p2} \tag{3.1.5}$$

Here (Fig.2.2.2, Fig.2.2.1):

$$[M_{2d_2}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi_2 & \sin\phi_2 & 0 \\ 0 & -\sin\phi_2 & \cos\phi_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.1.6}$$

$$[M_{d_2m_2}] = \begin{bmatrix} \cos\gamma_{m2} & 0 & \sin\gamma_{m2} & -X_{B2}\sin\gamma_{m2} \\ 0 & 1 & 0 & 0 \\ -\sin\gamma_{m2} & 0 & \cos\gamma_{m2} & -X_{B2}\cos\gamma_{m2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.1.7}$$

$$[M_{c_2p_2}] = \begin{bmatrix} 1 & 0 & 0 & S_{r2}\cos q_2 \\ 0 & 1 & 0 & S_{r2}\sin q_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.1.8}$$

$$[M_{m_2c_2}] = \begin{bmatrix} \cos\phi_p & -\sin\phi_p & 0 & 0 \\ \sin\phi_p & \cos\phi_p & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.1.9}$$

The machine root angle γ_{m2} in equation (3.17) is equal to gear root cone angle γ.

Step 3: The derivation of the equation of meshing is based on the equation

$$\vec{n}_{m2} \cdot \vec{v}_{m2}^{(p2)} = 0 \tag{3.1.10}$$

The subscript "$m2$" means that vectors in equation (3.1.10) are represented in coordinate system S_{m2} ; $\vec{n}_{m2}$ is the unit normal to the generating surface; $\vec{v}_{m2}^{(p2)} = \vec{v}_{m2}^{(p)} - \vec{v}_{m2}^{(2)}$ is the relative (sliding) velocity. Vector $\vec{n}_{m2}$ is represented by the matrix equation

$$\vec{n}_{m2} = [L_{m_2p_2}]\vec{n}_{p2} = \begin{bmatrix} -\cos\alpha_G\cos(\theta_G + \phi_p) \\ -\cos\alpha_G\sin(\theta_G + \phi_p) \\ \sin\alpha_G \end{bmatrix} \tag{3.1.11}$$

where $[L_{m_2p_2}]$ is the 3×3 submatrix $[M_{m_2p_2}]$.

We consider that the axes of rotation of the cradle and the gear intersect each other (Fig.2.2.2(a)), thus

$$\vec{v}_{m2}^{(p2)} = (\vec{\omega}_{m2}^{(p)} - \vec{\omega}_{m2}^{(2)}) \times \vec{r}_{m_2} = \vec{\omega}_{m2}^{(p2)} \times \vec{r}_{m_2} \tag{3.1.12}$$

where

$$\vec{\omega}_{m2}^{(p2)} = [-\cos\gamma_2 \quad 0 \quad (\frac{1}{R_{aG}} - \sin\gamma_2)]^T \tag{3.1.13}$$

We assumed that $|\vec{\omega}_{m2}^{(p)}| = 1$ in equation (3.1.13). Equations from (3.1.10) to (3.1.13) yield the following relation

$$s_G = \frac{A(\theta_G, \phi_p)}{B(\theta_G, \phi_p)} \tag{3.1.14}$$

Here

$$\begin{aligned} A(\theta_G, \phi_p) \quad = \quad & n_{m_2x}[-A_1(\sin\gamma_2 - \frac{1}{R_{aG}})] + n_{m_2y}[X_{B_2}\cos\gamma_2 + A_2(\sin\gamma_2 - \frac{1}{R_{aG}})] \\ & + n_{m_2z}A_1\cos\gamma_2 \end{aligned} \tag{3.1.15}$$

$$\begin{aligned} B(\theta_G, \phi_p) \quad = \quad & -n_{m_2x}\sin\alpha_G\sin(\theta_G + \phi_p) + n_{m_2y}[(\sin\gamma_2 - \frac{1}{R_{aG}})\sin\alpha_G\cos(\theta_G + \phi_p) \\ & - \cos\alpha_G\cos\gamma_2] + n_{m_2z}\cos\gamma_2\sin\alpha_G\sin(\theta_G + \phi_p) \end{aligned} \tag{3.1.16}$$

$$A_1 \quad = \quad r_c\sin(\theta_G + \phi_p) - S_{r2}\sin(q_2 - \phi_p) \tag{3.1.17}$$

$$A_2 \quad = \quad r_c\cos(\theta_G + \phi_p) - S_{r2}\cos(q_2 - \phi_p) \tag{3.1.18}$$

Step 4: Equations (3.1.5) and (3.1.14) considered simultaneously represent the gear surface in three- parametric form but with related parameters. Since parameter s_G in equation of meshing

(3.1.14) is linear, it can be eliminated in equation (3.1.5), and then the gear tooth surface will be represented in two-parametric form, by the vector function $\vec{r}_2(\theta_G, \phi_p)$.

3.2 Mean Contact Point and Gear Principal Directions and Curvatures

The mean contact point M is shown in Fig.2.3.1. Usually, M is chosen in the middle of the tooth surface. The gear tooth surface and the pinion tooth surface must contact each other at M.

The procedure of local synthesis discussed in section 2.1 is directed at providing improved conditions of meshing and contact at M and in the neighborhood of M. The location of point M is determined with parameters XL and RL (Fig.2.3.1) that are represented by the following equations

$$XL = A_m \cos\Gamma_2 - (b_G - \frac{h_m + c}{2}) \sin\Gamma_2 \tag{3.2.1}$$

$$RL = A_m \sin\Gamma_2 - (b_G - \frac{h_m - c}{2}) \cos\Gamma_2 \tag{3.2.2}$$

Here: A_m is the pitch cone mean distance; h_m is the mean whole depth; b_G is the gear mean dedendum; c is the clearance Equations (3.2.1), (3.2.2) and vector equation $\vec{r}_2(\theta_G, \phi_p)$ for the gear tooth surface allows to determine the surface parameters θ_G^* and ϕ_p^* for the mean contact point from the equations

$$X_2(\theta_G^*, \phi_p^*) = XL \tag{3.2.3}$$

$$Y_2^2(\theta_G^*, \phi_p^*) + Z_2^2(\theta_G^*, \phi_p^*) = (RL)^2 \tag{3.2.4}$$

Gear Principal Directions and Curvatures

The gear principal directions and curvatures can be expressed in terms of principal curvatures and directions of the generating surface (see chapter (13) in [4]), that is the cone surface.

Step 1: The cone principal directions are represented in S_{p2} by the equations (see (3.1.1))

$$\vec{e}^{(p)}_{sp_2} = \frac{\frac{\partial \vec{r}_{p_2}}{\partial \theta_G}}{\left|\frac{\partial \vec{r}_{p_2}}{\partial \theta_G}\right|} = [-\sin\theta_G \qquad \cos\theta_G \qquad 0]^T \tag{3.2.5}$$

$$\vec{e}^{(p)}_{qp_2} = \frac{\frac{\partial \vec{r}_{p_2}}{\partial s_G}}{\left|\frac{\partial \vec{r}_{p_2}}{\partial s_G}\right|} = [-\sin\alpha_G\cos\theta_G \qquad -\sin\alpha_G\sin\theta_G \qquad -\cos\alpha_G]^T \tag{3.2.6}$$

The superscript "p" indicates that the cone surface Σ_p is considered. Unit vector $\vec{e}^{(p)}_{qp_2}$ is directed along the cone generatrix and unit vector $\vec{e}^{(p)}_{sp_2}$ is perpendicular to $\vec{e}^{(p)}_{qp_2}$. The unit vectors of cone principal directions are represented in S_{m2} by the equations

$$\vec{e}^{(p)}_{sm_2} = [-\sin(\theta_G + \phi_p) \qquad \cos(\theta_G + \phi_p) \qquad 0]^T \tag{3.2.7}$$

$$\vec{e}^{(p)}_{qm_2} = [-\sin\alpha_G\cos(\theta_G + \phi_p) \qquad -\sin\alpha_G\sin(\theta_G + \phi_p) \qquad -\cos\alpha_G]^T \tag{3.2.8}$$

The cone principal curvatures are:

$$\kappa^{(p)}_s = \frac{\cos\alpha_G}{r_c - s_G\sin\alpha_G} \qquad \text{and} \qquad \kappa^{(p)}_q = 0 \tag{3.2.9}$$

Step 2: The determination of principal curvatures and directions for gear tooth surface Σ_2 is based on equations from (1.2.6) to (1.2.8). The superscript "2" in these equations must be changed

for "p" and superscript "1" for "2". The second derivative of cutting ratio, $m'_{21} \equiv m'_{p2}$ is zero because the cutting ratio is constant. The principal curvatures of the gear tooth surface will be determined as κ_f and κ_h. The principal directions on gear tooth surface will be represented in by $\vec{e}_f$ and $\vec{e}_h$ and they can be determined from equations (1.2.9) and (1.2.10). To represent in S_2 the principal directions on gear tooth surface Σ_2 and its unit normal we use the matrix equation that describe the coordinate transformation from S_{m2} to S_2. This equation is

$$\vec{a}_2 = [L_{2d_2}][L_{d_2m_2}]\vec{a}_{m2} \tag{3.2.10}$$

Here: $\vec{a}_{m2}$ stands for vectors $\vec{n}_{m2}$, $\vec{e}_{fm_2}$ and $\vec{e}_{hm_2}$, and $\vec{a}_2$ stands for $\vec{n}_2$, $\vec{e}^{(2)}_{I2}$ and $\vec{e}^{(2)}_{II2}$.

4 Local Synthesis of Spiral Bevel Gears

4.1 Conditions of Synthesis

The basic principles of local synthesis of gear tooth surfaces discussed in Section 1 will enable us to determine the principle curvatures and directions of the being synthesized pinion. Thus, we will be able to determine the required machine-tool settings for the pinion. While solving the problem of local synthesis, we will consider as known:

(i) The location of the mean contact point M in a fixed coordinate system, and the orientation of the normal to gear surface Σ_2.

(ii) The principle curvatures and directions on Σ_2 at M. The local synthesis of gear tooth surfaces must satisfy the following requirements:

(1) The pinion and gear tooth surfaces must be in contact at M.

(2) The tangent to the contact path on the gear tooth surface must be of the prescribed direction.

(3) Function of gear ratio $m_{21}(\phi_1)$ in the neighborhood of mean contact point must be a linear one, be of prescribed value at M and have the prescribed value for the derivative $m'_{21}(\phi_1)$ at M. The satisfaction of these requirements provides a parabolic type of function for transmission errors of the desired value at each cycle of meshing.

(4) The major axis of the instantaneous contact ellipse must be of the desired value (with the given elastic approach of tooth surfaces).

4.2 Procedure of Synthesis

We will consider in this section the following steps of the computational procedure: (i) representation of gear mean contact point in a fixed coordinate system S_h ; (ii) satisfication of equation of meshing of the pinion and gear at the mean contact point ; (iii) representation of principle directions on gear tooth surface Σ_2 in S_h; (iv) observation of the desired derivative $m'_{21}(\phi_1)$. (v) observation at the mean contact point of the desired direction of the tangent to the path contact on gear tooth

surface ; (vi) observation at the mean contact point of the desired length of the major axis of the contact ellipse; (vi) determination of principal directions and curvatures on pinion tooth surface Σ_1 at the mean contact point.

Step 1: We set up a fixed coordinate system S_h that is rigidly connected to the gear mesh housing (Fig.4.2.1(a)). In addition to S_h, we will use coordinate systems S_2 (Fig.4.2.1(a)) and S_1 (Fig.4.2.1(b)) that are rigidly connected to gears 2 and 1, respectively. We designate with ϕ_2' and ϕ_1' the angles of rotation of gears being in mesh. We have to emphasize that with this designation $\phi_i'(i = 1, 2)$ we differentiate the angle of gear rotation in meshing from the angle ϕ_i of gear rotation in the process of generation.

The orientation of coordinate system S_h is based on following considerations: (i) The axes of rotation of the pinion and the gear in a drive of spiral bevel gears intersect each other. Taking into account the possible gear misalignment, we will consider that the pinion-gear axes are crossed at angle Γ and the shortest distance is E. (ii) We will choose that X_h coincides with the pinion axis and O_h is located on the shortest distance (Fig.4.2.1(a)). (iii) Considering as given the shaft angle Γ, we will define $\vec{j}_h$– the unit vector of Y_h – as follows

$$\vec{j}_h = \frac{\vec{i}_h \times \vec{a}_h}{|\vec{i}_h \times \vec{a}_h|} \tag{4.2.1}$$

where $\vec{a}_h$ is the unit vector of gear axis that is parallel to plane (X_h, Y_h).

The coordinate transformation from S_2 to S_h is based on matrix equation

$$\vec{r}_h^{(M)} = [M_{hd}][M_{d2}]\vec{r}_2(\theta_G, \phi_p) \tag{4.2.2}$$

where S_d (Fig.4.2.1) is an auxiliary fixed coordinate system. The unit normal to Σ_2 is repre-

sented in S_h as

$$\vec{n}_h^{(2)} = [L_{hd}][L_{d2}]\vec{n}_2(\theta_G, \phi_p) \tag{4.2.3}$$

Here (Fig. 4.2.1)

$$[M_{d2}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\cos\phi_2' & \sin\phi_2' & 0 \\ 0 & -\sin\phi_2' & -\cos\phi_2' & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.2.4}$$

$$[M_{hd}] = \begin{bmatrix} \cos\Gamma & 0 & \sin\Gamma & 0 \\ 0 & 1 & 0 & E \\ -\sin\Gamma & 0 & \cos\Gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.2.5}$$

where Γ is the shaft angle.

Equations (4.2.3), (4.2.2) and (4.2.3) enable to represent in S_h the position vector and unit contact normal at M by

$$\vec{r}_h^{(1)}(\theta_G^*, \phi_p^*, \phi_2') \qquad \vec{n}_h^{(2)}(\theta_G^*, \phi_p^*, \phi_2') \tag{4.2.6}$$

where (θ_G^*, ϕ_p^*) are the surface coordinates for the mean contact point at Σ_2 ; the angle ϕ_2' of rotation of gear 2 will be determined from the equation of meshing (see below).

Step 2: The equation of meshing of pinion and gear at the mean contact point is

$$\vec{n}_h^{(2)} \cdot \vec{v}_h^{(12)} = f(\theta_G^*, \phi_p^*, \phi_2') = 0 \qquad (4.2.7)$$

Here (Fig. 4.2.1)

$$\vec{v}_h^{(12)} = [(\vec{\omega}_h^{(1)} - \vec{\omega}_h^{(2)}) \times \vec{r}_h^{(M)}] - (\vec{E} \times \omega_h^{(2)}) \qquad (4.2.8)$$

$$\vec{\omega}_h^{(1)} = [-1 \quad 0 \quad 0]^T \qquad (|\vec{\omega}_h^{(1)}| = 1) \qquad (4.2.9)$$

$$\vec{\omega}_h^{(2)} = \frac{N_1}{N_2}[\cos\Gamma \quad 0 \quad -\sin\Gamma]^T \qquad (4.2.10)$$

since at point M the angular velocity ratio is

$$\frac{\omega^{(2)}}{\omega^{(1)}} = \frac{N_1}{N_2} \qquad (4.2.11)$$

Substituting equations (4.2.3), (4.2.8)- (4.2.11) in equation (4.2.7), we can solve equation (4.2.7) for ϕ_2'. Usually equation (4.2.7) yields two solutions for ϕ_2' but the smaller one, say $(\phi_2')^*$, should be chosen.

Step 3: We consider as known the principal curvatures and directions on Σ_2 at any point of Σ_2, including the mean contact point (see section 3). To represent in S_h the principal directions at the mean contact point, we use the matrix equation

$$\vec{a}_h^{(M)} = [L_{hd}][L_{d2}]\vec{a}_2 \qquad (4.2.12)$$

where $\vec{a}_2$ is the unit vector of principal directions on Σ_2 that is represented in S_2. The following steps of computational procedure are exactly the same that have been described in section 1.2. This procedure permit determination of the pinion principal directions and curvatures at the mean contact point.

5 Pinion Machine-Tool Settings

5.1 Introduction

We consider at this stage of investigation as known:

(i) the common position vector $\vec{r}_h^{(i)}$ and unit normal $\vec{n}_h^{(i)}$ at the point of contact point M of Σ_2 and Σ_1

(ii) pinion surface principal directions and curvatures at M.

The goal is to determine the settings of the pinion and the head- cutter that will satisfy the conditions of local synthesis. We consider that the pinion surface and the generating surface are in line contact. Henceforth, we will consider two types of the generating surface: (a) a cone surface, and (b) a surface of revolution. We consider that each side of the pinion tooth is generated separately and two head-cutters must be applied for the pinion generation.

5.2 Head-Cutter Surface

Cone Surface

The cone surface is generated by straight blades being rotated about the z_F-axis (Fig. 5.2.1(a)). The Σ_F equations are represented in coordinate system S_F that is rigidly connected to the head-cutter as following:

$$\vec{r}_F = \begin{bmatrix} (R_{cp} + s_F \sin\alpha_F)\cos\theta_F \\ (R_{cp} + s_F \sin\alpha_F)\sin\theta_F \\ -s_F\cos\alpha_F \\ 1 \end{bmatrix} \tag{5.2.1}$$

Here: s_F and θ_F are the surface coordinates; α_F and R_{cp} are the blade angle and the radius of the cone in plane $z_F = 0$. The blade angle α_F is standardized and is considered as known. Parameter

α_F is considered as negative for the pinion convex side and α_F is positive for the pinion concave side. The point radius R_{cp} is considered as unknown and must be determined later.

The unit normal to pinion tooth surface is represented as

$$\vec{n}_F = \frac{\vec{N}_F}{|\vec{N}_F|} \qquad \text{and} \qquad \vec{N}_F = \frac{\partial \vec{r}_F}{\partial \theta_F} \times \frac{\partial \vec{r}_F}{\partial s_F} \tag{5.2.2}$$

i.e.,

$$\vec{n}_F = -[\cos\alpha_F \cos\theta_F \quad \cos\alpha_F \sin\theta_F \quad \sin\alpha_F]^T \tag{5.2.3}$$

The principal directions on the cone surface are:

$$\vec{e}_I^{(F)} = \frac{\frac{\partial \vec{r}_F}{\partial \theta_F}}{\left|\frac{\partial \vec{r}_F}{\partial \theta_F}\right|} = [-\sin\theta_F \quad \cos\theta_F \quad 0]^T \tag{5.2.4}$$

$$\vec{e}_{II}^{(F)} = \frac{\frac{\partial \vec{r}_F}{\partial s_F}}{\left|\frac{\partial \vec{r}_F}{\partial s_F}\right|} = [\sin\alpha_F \cos\theta_F \quad \sin\alpha_F \sin\theta_F \quad -\cos\alpha_F]^T \tag{5.2.5}$$

The corresponding principal curvatures are

$$\kappa_I^{(F)} = \frac{\cos\alpha_F}{R_{cp} + s_F \sin\alpha_F} \qquad \text{and} \qquad \kappa_{II}^{(F)} = \frac{1}{\rho} \tag{5.2.6}$$

Surface of Revolution

We consider that the head-cutter surface Σ_F is generated by a circular arc of radius ρ by rotation about the z_0-axis that coincides with the z_F-axis of the head-cutter (Fig. 5.2.1(b)) and (Fig.5.2.1(c)). The shape of the blade is represented in S_o by the vector equations

$$\overline{O_oN} = \overline{O_oC} + \overline{CN} = (X_o^{(c)} + \rho\cos\lambda)\vec{i}_o + (Z_o^{(c)} + \rho\sin\lambda)\vec{k}_o \tag{5.2.7}$$

Here: $(X_0^{(c)}, Z_0^{(c)})$ are algebraic values that represent in S_o the location of center C of the arc; $\rho = |\overline{CN}|$ is the radius of the circular arc and is an algebraic value, ρ is positive when center C is on the positive side of the unit normal. ; λ is the independent variable that determines the location of the current point N of the arc. By using the coordinate transformation from S_o to S_F (Fig.5.2.1(c)), we obtain the following equations of the surface of the head-cutter:

$$\vec{r}_F = \begin{bmatrix} (X_o^{(c)} + \rho\cos\lambda)\cos\theta_F \\ (X_o^{(c)} + \rho\cos\lambda)\sin\theta_F \\ Z_o^{(c)} + \rho\sin\lambda \\ 1 \end{bmatrix} \tag{5.2.8}$$

where λ and θ_F are the surface coordinates (independent variables).

The surface unit normal $\vec{n}_F$ is represented by the following equations

$$\vec{n}_F = \frac{\vec{N}_F}{|\vec{N}_F|} \qquad \text{and} \qquad \vec{N}_F = -\frac{\partial\vec{r}_F}{\partial\theta_F} \times \frac{\partial\vec{r}_F}{\partial\lambda} \tag{5.2.9}$$

Then we obtain

$$\vec{n}_F = -[\cos\lambda\cos\theta_F \quad \cos\lambda\sin\theta_F \quad \sin\lambda]^T \tag{5.2.10}$$

The variable λ at the mean contact point M has the same value as the standardized blade angle α_F. The principal directions on the head-cutter surface are

$$\vec{e}_I^{F} = \frac{\dfrac{\partial \vec{r}_F}{\partial \theta_F}}{\left|\dfrac{\partial \vec{r}_F}{\partial \theta_F}\right|} = [-\sin\theta_F \quad \cos\theta_F \quad 0]^T \tag{5.2.11}$$

$$\vec{e}_{II}^{(F)} = -\frac{\dfrac{\partial \vec{r}_F}{\partial \lambda}}{\left|\dfrac{\partial \vec{r}_F}{\partial \lambda}\right|} = [\sin\lambda\cos\theta_F \quad \sin\lambda\sin\theta_F \quad -\cos\lambda]^T \tag{5.2.12}$$

The principal curvatures are

$$\kappa_I^{(F)} = \frac{\cos\lambda}{X_o^{(c)} + \rho\cos\lambda} \quad \text{and} \quad \kappa_{II}^{(F)} = \frac{1}{\rho} \tag{5.2.13}$$

The radius R_{cp} of the head-cutter in plane (Fig. 5.2.1) can be determined from the equations

$$R_{cp} = X_o^{(c)} + \rho\sqrt{1 - \left(\frac{Z_0^{(c)}}{\rho}\right)^2} \tag{5.2.14}$$

5.3 Observation of a Common Normal at the Mean Contact Point for Surfaces Σ_p, Σ_2, Σ_F and Σ_1

We consider that at the mean contact point M four surfaces- $\Sigma_p, \Sigma_2, \Sigma_F$ and Σ_1- must be in tangency. The contact of Σ_p and Σ_2 at M has been already provided due to the satisfication of

their equation of meshing (3.1.10). Our goal is to determine the conditions for the coincidence at M of the unit normals to Σ_F, Σ_p and Σ_2. The tangency of Σ_1 with the three above mentioned surfaces will be discussed below.

We will consider the coincidence of the unit normals in coordinate system S_{m1}. To determine the orientation of coordinate system S_h with respect to S_{m1}, let us imagine that the set of coordinate systems S_h, S_1 and S_2 (Fig.4.2.1) with gears 1 and 2 is installed in S_{m1} with observation of following conditions (Fig.5.2.2): (i) axis x_h of S_h coincides with axis x_p of S_p; (ii) coordinate system S_1 coincides with S_p and the orientation of S_h with respect to S_1 is designated with angle $\phi_h = (\phi_1^{'})^0$ where ϕ_h is the to be determined instalment angle. Angle ϕ_h will be determined from the conditions of coincidence of the unit normals to Σ_F, Σ_2 Σ_p and Σ_1. The procedure for derivation is as follows:

Step 1: Consider that the coordinate system S_h with the point of tangency of surfaces Σ_2 and Σ_p is installed in S_{m1}. We may represent the surface unit normal $\vec{n}^{(2)}$ in S_{m1} by using the following matrix equation (Fig. 5.2.2).

$$\vec{n}_{m1}^{(2)} = [L_{m_1p}][L_{ph}]\vec{n}_h^{(2)} = \begin{bmatrix} \cos\gamma_1 & 0 & -\sin\gamma_1 \\ 0 & 1 & 0 \\ \sin\gamma_1 & 0 & \cos\gamma_1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_h & -\sin\phi_h \\ 0 & \sin\phi_h & \cos\phi_h \end{bmatrix} \vec{n}_h^{(M)} \tag{5.3.1}$$

The unit vector $\vec{n}_h^{(2)}$ has been represented by equation (4.2.3).

Step 2: The unit vector to the surface of Σ_F of the head-cutter that generates the pinion has been represented in S_F by equation (5.2.3) for a cone and equation (5.2.10) for a surface of revolution. Axes of coordinate systems S_F and S_{m1} have the same orientation and

$$\vec{n}_{m1}^{(F)} = \vec{n}_F^{(F)} \tag{5.3.2}$$

Equations (5.3.1), (5.3.2), (5.2.3) and (5.2.10) yield the following equations

$$\cos\theta_F^* = -\frac{n_{xh}^{(2)} + \sin\alpha_F \sin\gamma_1}{\cos\gamma_1 \cos\alpha_F} \tag{5.3.3}$$

$$\cos\phi_h = \frac{a_1 n_{xh}^{(2)} + a_2 n_{zh}^{(2)}}{(n_{yh}^{(2)})^2 + (n_{zh}^{(2)})^2} \qquad \sin\phi_h = \frac{a_1 n_{yh}^{(2)} - a_2 n_{zh}^{(2)}}{(n_{yh}^{(2)})^2 + (n_{zh}^{(2)})^2} \tag{5.3.4}$$

Here:

$$a_1 = -\cos\alpha_F \sin\theta_F^* \qquad a_2 = \cos\alpha_F \sin\gamma_1 \cos\theta_F^* - \sin\alpha_F \cos\gamma_1 \tag{5.3.5}$$

The advantage of the proposed approach is that the coincidence of the unit normals to surfaces $\Sigma_F, \Sigma_2, \Sigma_p$ and Σ_1 can be achieved with standard blade angles and without a tilt of the head-cutter.

5.4 Basic Equations for Determination of Pinion Machine-Tool Settings

At this stage of investigation we will consider as known: $\kappa_I^{(1)}, \kappa_{II}^{(1)}, \vec{e}_{Im1}^{(1)}, \vec{e}_{IIm1}^{(1)}, \vec{n}_{m1}^{M}$ and $\vec{r}_{m1}^{M}$. It is necessary to determine: $\kappa_I^{(F)}, \kappa_{II}^{(F)}, \sigma^{(1F)}$, R_{cp}, E_{m1}, X_{BG}, and m'_{F1}. Here: $\kappa_I^{(1)}, \kappa_{II}^{(1)}, \vec{e}_{Im1}^{(1)}$ and $\vec{e}_{IIm1}^{(1)}$ are the principal curvatures and unit vectors of principal directions on the pinion surface that are taken at mean contact point M; $\vec{r}_{m1}^{(M)}$ and $\vec{n}_{m1}^{(M)}$ are the position vector of M and the contact normal at M. The subscript "m1" indicates that the vectors are represented in S_{m1}. Designations $\kappa_I^{(F)}$ and $\kappa_{II}^{(F)}$ indicate the principal curvatures of the surface of the pinion head-cutter that are taken at M. The angle $\sigma^{(1F)}$ is formed by the unit vectors $\vec{e}_I^{(1)}$ and $\vec{e}_I^{(F)}$ of principal directions on Σ_1 and Σ_F; R_{cp} is the cutter "point radius" (Fig.5.5.1) that is measured in plane $z_F = 0$ and is dependent

on $\kappa_I^{(1)}$. E_{m1} and X_{G1} are the pinion settings for its generation (Fig.2.1.1 and Fig.2.1.2); m_{F1}, which is equal to $\frac{1}{R_{cp}}$, and m'_{F1} are the cutting ratio and its derivative.

We recall that the pinion surface curvatures $\kappa_I^{(1)}$ and $\kappa_{II}^{(1)}$ have been determined in the process of local synthesis. Vectors $\vec{e}_{Ih}^{(1)}, \vec{e}_{IIh}^{(1)}, \vec{r}_h^{(M)}$ have been determined in system S_h. To represent these vectors in S_{m1} we have to apply the coordinate transformation from S_h to S_{m1} similar to equation (5.3.1).

$$\vec{a}_{m1} = [L_{m_1p}][L_{ph}]\vec{a}_h \tag{5.4.1}$$

where $\vec{a}_h$ represents that principal directions of the pinion surface $\vec{e}_{Ih}^{(1)}$ and $\vec{e}_{IIh}^{(1)}$, the position vector of mean contact point $\vec{r}_h^{(M)}$; $\vec{a}_{m1}$ represents the corresponding vectors $\vec{e}_{Im1}^{(1)}, \vec{e}_{IIm1}^{(1)}$ and $\vec{r}_{m1}^{(m)}$.

Now our goal, as it was mentioned above, is to determine $\kappa_I^{(F)}, \kappa_{II}^{(F)}, \sigma^{(1F)}$, E_{m1}, X_{G1}, m_{F1} and m'_{F1}. We recall that vectors $\vec{e}_I^{(1)}$ and $\vec{e}_{II}^{(1)}$ are known from the local synthesis, and $\vec{e}_I^{(F)}$ and $\vec{e}_{II}^{(F)}$ become known from equations (5.2.4) and (5.2.5) for straight blade, and from equations (5.2.11) and (5.2.12) for curved blade, after the coincidence of the contact normal to surfaces Σ_2, Σ_p and Σ_F is provided. Thus parameter $\sigma^{(1F)}$ can be determined from the equations

$$\left.\begin{aligned} \sin\sigma^{(1F)} &= \vec{n}_{m1}^{(M)} \cdot (\vec{e}_{Im1}^{(1)} \times \vec{e}_{Im1}^{(F)}) \\ \cos\sigma^{(1F)} &= \vec{e}_{Im1}^{(1)} \cdot \vec{e}_{Im1}^{(F)} \end{aligned}\right\} \tag{5.4.2}$$

According to Fig. 5.5.1, since the Z_{m1}-axis is parallel to Z_F-axis, surface parameter s_F for the cone surface at mean point can be determined as:

$$s_F^* = \frac{|Z_{m1}|}{\cos\alpha_F} \tag{5.4.3}$$

Parameter $\kappa_{II}^{(F)}$ is equal to zero for a cone surface of the head-cutter and it must be chosen for a head-cutter with a surface of revolution. Then, the number of remaining parameters to-be determined becomes equal to five and they are: $\kappa_I^{(F)}, E_{m1}, X_{G1}, m_{1F}$ and m'_{F1}.

It will be shown below that we can derive only four equations for determination of the unknowns of the output data. Therefore one more parameter has to be chosen, and this is m'_{F1}–the modified roll. Usually, it is sufficient to choose $m'_{F1} = 0$, but the more general case with $m'_{F1} \neq 0$ is considered in this report as well.

The to be derived equations are as follows,

$$\vec{n}_{m1}^{(M)} \cdot \vec{v}_{m1}^{(1F)} = 0 \tag{5.4.4}$$

$$a_{11}a_{22} = a_{12}^2 \tag{5.4.5}$$

$$a_{11}a_{23} = a_{12}a_{13} \tag{5.4.6}$$

$$a_{12}a_{33} = a_{13}a_{23} \tag{5.4.7}$$

Equation (5.4.4) is the equation of meshing of the pinion and head-cutter that is applied at the mean contact point. Equations from (5.4.5) to (5.4.7) come from the conditions of existence of instantaneous line contact between Σ_1 and Σ_F. The coefficients a_{ij} in equation (5.4.5)-(5.4.7) are represented as follows,

$$a_{11} = \kappa_I^{(F)} - \kappa_I^{(1)} \cos^2 \sigma^{(1F)} - \kappa_{II}^{(1)} \sin^2 \sigma^{(1F)} \tag{5.4.8}$$

$$a_{12} = a_{21} = \frac{\kappa_I^{(1)} - \kappa_{II}^{(1)}}{2} \sin 2\sigma^{(1F)} \tag{5.4.9}$$

$$a_{13} = a_{31} = -\kappa_I^{(F)} v_I^{(1F)} - \left[\vec{\omega}^{(1F)} \; \vec{n} \; \vec{e}_I^{(F)}\right] \tag{5.4.10}$$

$$a_{22} = \kappa_{II}^{(F)} - \kappa_I^{(1)} \sin^2 \sigma^{(1F)} - \kappa_{II}^{(1)} \cos^2 \sigma^{(1F)} \tag{5.4.11}$$

$$a_{23} = a_{32} = -\kappa_{II}^{(F)} v_{II}^{(1F)} - \left[\vec{\omega}^{(1F)} \; \vec{n} \; \vec{e}_{II}^{(F)}\right] \tag{5.4.12}$$

$$\begin{aligned} a_{33} = \; & \kappa_I^{(F)} \left(v_I^{(1F)}\right)^2 + \kappa_{II}^{(F)} \left(v_{II}^{(1F)}\right)^2 - \left[\vec{n} \; \vec{\omega}^{(1F)} \; \vec{v}^{(1F)}\right] \\ & - \left[\vec{n}_{m1} \; \vec{\omega}^{(1)} \; \vec{v}_{tr}^{(F)}\right] + \left[\vec{n}_{m1} \; \vec{\omega}_{m1}^{(F)} \; \vec{v}_{tr}^{(1)}\right] + \frac{\left(\vec{\omega}^{(1)}\right)^2}{\vec{\omega}^{(2)}} m'_{F1} \left(\vec{n} \cdot \vec{v}_{tr}^{(2)}\right) \end{aligned} \tag{5.4.13}$$

Vectors in equation of meshing (5.4.4) can be represented as follows

$$\vec{\omega}_{m1}^{(1)} = [\cos \gamma_1 \quad 0 \quad \sin \gamma_1]^T \qquad (|\vec{\omega}^{(1)}| = 1) \tag{5.4.14}$$

$$\vec{\omega}_{m1}^{(F)} = \frac{1}{R_{ap}} [0 \;\; 0 \;\; 1] \tag{5.4.15}$$

where R_{ap}, which is equal to $\dfrac{1}{m_{F1}}$, is the ratio of roll.

$$\vec{\omega}_{m1}^{(1F)} = \vec{\omega}_{m1}^{(1)} - \vec{\omega}_{m1}^{(F)} \tag{5.4.16}$$

$$\vec{v}_{m1}^{(1F)} = \vec{v}_{tr}^{(1)} - \vec{v}_{tr}^{(F)} \tag{5.4.17}$$

$$\vec{v}_{tr}^{(1)} = \vec{\omega}_{m1}^{(1)} \times \vec{r}_{m1}^{(M)} = \begin{bmatrix} -Y_{m1} \sin\gamma_1 \\ X_{m1} \sin\gamma_1 - Z_{m1} \cos\gamma_1 \\ Y_{m1} \cos\gamma_1 \end{bmatrix} \tag{5.4.18}$$

$$\vec{v}_{tr}^{(F)} = \vec{\omega}_{m1}^{(F)} \times \left(\vec{r}_{m1}^{(M)} - \overline{O_F O_1}\right) = \begin{bmatrix} -Y_{m1} m_{F1} + E_{m1} m_{F1} \\ X_{m1} m_{F1} + m_{F1} X_{G1} \cos\gamma_1 \\ 0 \end{bmatrix} \tag{5.4.19}$$

5.5 Determination of Cutter Point Radius

Step.1: Equations (5.4.3), (5.4.6) and (5.4.7) yield the following expression for $\kappa_I^{(F)}$

$$\kappa_I^{(F)} = \frac{\kappa_I^{(1)} \kappa_{II}^{(1)} + \kappa_{II}^{(F)} (\kappa_I^{(1)} \cos^2 \sigma^{(1F)} + \kappa_{II}^{(1)} \sin^2 \sigma^{(1F)})}{\kappa_{II}^{(F)} - \kappa_I^{(1)} \sin^2 \sigma^{(1F)} - \kappa_{II}^{(1)} \cos^2 \sigma^{(1F)}} \tag{5.5.1}$$

Step 2: According to *Meusnier's theorem*, the cutter radius R_m at the mean contact point is (Fig.5.5.1)

$$R_m = \frac{\cos\alpha_F}{|\kappa_I^{(F)}|} \tag{5.5.2}$$

As shown in Fig.5.2.1. the cutter point radius can be determined for a straight blade cutter as follows,

$$R_{cp} - R_m - s_F^* \sin\alpha_F \tag{5.5.3}$$

For the arc blade, the location of the center of the arc can be determined in S_o by following equations,

$$X_o^{(c)} = R_m - \rho \cos \alpha_F \tag{5.5.4}$$

$$Z_o^{(c)} = R_m - \rho \sin \alpha_F \tag{5.5.5}$$

Knowing $X_o^{(c)}$ and $Z_o^{(c)}$, we can determine the point radius for the arc blade by equation (5.2.14).

In order to find the position vector of the center of the head cutter, we define the following two vectors in S_{m1} as shown in Fig. 5.5.1.

$$\vec{\rho}^{(o)} = \vec{n} \cos \alpha_F - \vec{e}_{II}^{(F)} \sin \alpha_F \tag{5.5.6}$$

$$\vec{\rho}^{(c)} = \begin{bmatrix} \cos\theta_F & -\sin\theta_F & 0 & 0 \\ \sin\theta_F & \cos\theta_F & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_o^{(c)} \\ 0 \\ Z_o^{(c)} \\ 1 \end{bmatrix} = \begin{bmatrix} X_o^{(c)} \cos\theta_F \\ X_o^{(c)} \sin\theta_F \\ Z_o^{(c)} \\ 1 \end{bmatrix} \tag{5.5.7}$$

where, $\rho^{(o)}$ is a unit vector directed from the blade tip M_o to the cutter center O_F, and $\rho^{(c)}$ is a position vector directed from O_F to the arc center C. Referring to Fig.5.2.2 and Fig.5.5.2, the position vector of the cutter center O_F with respect to O_h, $\vec{r}_h^{(O_F)}$ can be determined in system S_{m1} as follows,

For straight blade:

$$\vec{r}_h^{(O_F)} = \vec{r}_{m1}^{(M)} - S_F^* \vec{e}_{II}^{(F)} + R_{cp} \vec{\rho}^{(o)} \tag{5.5.8}$$

For arc blade :

$$\vec{r}_h^{(O_F)} = \vec{r}_{m1}^{(M)} + \rho\vec{n}_{m1} - \vec{\rho}^{(c)} \tag{5.5.9}$$

It can be verified that the Z_{m1} component of $\vec{r}_h^{(O_F)}$ is zero, since equations (5.4.3) and (5.5.5) are observed. It is worth to mention that θ_F^* and s_F^* are the surface coordinates where the contact is at mean point. The values of θ_F^* and s_F^* will serve as the initial guess in tooth contact analysis.

5.6 Determination of $m_{F1} \equiv \frac{1}{R_{ap}}$, E_{M1} and X_{G1}

The determination of cutting ratio R_{ap}, settings E_{m1} and X_{G1} is based on application of equations (5.4.2), (5.4.4) and (5.4.5).

Initial Derivations

It is obvious that equation of meshing (5.4.4) is satisfied at point M if the relative velocity $\vec{v}^{(F1)}$ lies in plane that is tangent to the contacting surfaces at M. Thus, if velocity $\vec{v}^{(F1)}$ satisfies the equation,

$$\vec{v}^{(F1)} = v_I^{(F1)}\vec{e}_I^{(F)} + v_{II}^{(F1)}\vec{e}_{II}^{(F)} \tag{5.6.1}$$

it means that equation of meshing (5.4.4) is also satisfied. Assuming that vectors of equation (5.6.1)) are represented in coordinate system S_{m1}, we obtain

$$\vec{v}_{m1}^{(F1)} = \begin{bmatrix} v_I^{(F1)} e_{Im1X}^{(F)} + v_{II}^{(F1)} e_{IIm1X}^{(F)} \\ v_I^{(F1)} e_{Im1Y}^{(F)} + v_{II}^{(F1)} e_{IIm1Y}^{(F)} \\ v_I^{(F1)} e_{Im1Z}^{(F)} + v_{II}^{(F1)} e_{IIm1Z}^{(F)} \end{bmatrix} \tag{5.6.2}$$

For further derivations we will use the following expressions for a_{13} and a_{23}.

$$\left.\begin{aligned} a_{13} &= \kappa_I^{(F)} v_I^{(F1)} + M_{11} t_1 + M_{12} \\ a_{23} &= \kappa_{II}^{(F)} v_{II}^{(F1)} + M_{21} t_1 + M_{22} \end{aligned}\right\} \tag{5.6.3}$$

Here,

$$M_{11} = n_{m1X} e_{Im1Y}^{(F)} - n_{m1Y} e_{Im1X}^{(F)}$$

$$M_{12} = -\cos\gamma_1 [n_{m1Y} e_{Im1Z}^{(F)} - -n_{m1Z} e_{Im1Y}^{(F)}]$$

$$M_{21} = n_{m1X} e_{IIm1Y}^{(F)} - n_{m1Y} e_{IIm1X}^{(F)}$$

$$M_{22} = -\cos\gamma_1 [n_{m1Y} e_{IIm1Z}^{(F)} - -n_{m1Z} e_{IIm1Y}^{(F)}] \tag{5.6.4}$$

$$t_1 = m_{F1} - \sin\gamma_1 \tag{5.6.5}$$

Using equations (5.6.2), (5.6.3) and (5.4.16), we obtain

$$v_I^{(F1)} e_{Im1Z}^{(F)} + v_{II}^{(F1)} e_{IIm1Z}^{(F)} + Y_{m1} \cos\gamma_1 = 0 \tag{5.6.6}$$

Following Derivations

Step 1: Expressions for $v_I^{(F1)}$ and $v_{II}^{(F1)}$. Equations (5.6.6) and (5.4.4) represent a system of two linear equation in unknowns $v_I^{(F1)}$ and $v_{II}^{(F1)}$. The solution of these equations for the unknowns yields:

$$v_I^{(F1)} = L_{21}t_1 + L_{22} \tag{5.6.7}$$

$$v_{II}^{(F1)} = L_{11}t_1 + L_{12} \tag{5.6.8}$$

Here,

$$L_{21} = \frac{\epsilon_{IIm1Z}^{(F)}(a_{11}M_{21} - a_{12}M_{11})}{a_{12}\kappa_I^{(F)}\epsilon_{IIm1Z}^{(F)} + a_{11}\kappa_{II}^{(F)}\epsilon_{Im1Z}^{(F)}} \tag{5.6.9}$$

$$L_{22} = \frac{\epsilon_{IIm1Z}^{(F)}(a_{11}M_{22} - a_{12}M_{12}) - a_{11}\kappa_{II}^{(F)}Y_{m1}\cos\gamma_1}{a_{12}\kappa_I^{(F)}\epsilon_{IIm1Z}^{(F)} + a_{11}\kappa_{II}^{(F)}\epsilon_{Im1Z}^{(F)}} \tag{5.6.10}$$

$$L_{11} = \frac{-\epsilon_{Im1Z}^{(F)}(a_{11}M_{21} - a_{12}M_{11})}{a_{12}\kappa_I^{(F)}\epsilon_{IIm1Z}^{(F)} + a_{11}\kappa_{II}^{(F)}\epsilon_{Im1Z}^{(F)}} \tag{5.6.11}$$

$$L_{12} = \frac{-\epsilon_{Im1Z}^{(F)}(a_{11}M_{22} - a_{12}M_{12}) - a_{11}\kappa_I^{(F)}Y_{m1}\cos\gamma_1}{a_{12}\kappa_I^{(F)}\epsilon_{IIm1Z}^{(F)} + a_{11}\kappa_{II}^{(F)}\epsilon_{Im1Z}^{(F)}} \tag{5.6.12}$$

Step 2: Expression for $\vec{v}^{(F1)}$

Substituting the above equation in equation (5.6.2) , we obtain

$$\vec{v}^{(F1)} = \begin{bmatrix} X_{11}t_1 + X_{12} \\ \\ X_{21}t_1 + X_{22} \\ \\ X_{31}t_1 + X_{32} \end{bmatrix} \tag{5.6.13}$$

Here:

$$\left.\begin{aligned} X_{11} &= L_{21}\epsilon^{(F)}_{Im1X} + L_{11}\epsilon^{(F)}_{IIm1X} \\ X_{12} &= L_{22}\epsilon^{(F)}_{Im1X} + L_{12}\epsilon^{(F)}_{IIm1X} \\ X_{21} &= L_{21}\epsilon^{(F)}_{Im1Y} + L_{11}\epsilon^{(F)}_{IIm1Y} \\ X_{22} &= L_{22}\epsilon^{(F)}_{Im1Y} + L_{12}\epsilon^{(F)}_{IIm1Y} \\ X_{31} &= L_{21}\epsilon^{(F)}_{Im1Z} + L_{11}\epsilon^{(F)}_{IIm1Z} \\ X_{32} &= L_{22}\epsilon^{(F)}_{Im1Z} + L_{12}\epsilon^{(F)}_{IIm1Z} \end{aligned}\right\} \tag{5.6.14}$$

<u>Step 3</u>: Expression for $\vec{v}^{(F1)}_{tr}$

Equations (5.4.16) and (5.4.18) yield

$$\vec{v}^{(F)}_{tr} = \begin{bmatrix} X_{11}t_1 + X_{13} \\ \\ X_{21}t_1 + X_{23} \\ \\ X_{31}t_1 + X_{33} \end{bmatrix} \tag{5.6.15}$$

Here,

$$\left.\begin{aligned} X_{13} &= X_{12} - Y_{m1}\sin\gamma \\ X_{23} &= X_{22} + X_{m1}\sin\gamma - Z_{m1}\cos\gamma \\ X_{33} &= X_{32} + Y_{m1}\cos\gamma \end{aligned}\right\} \tag{5.6.16}$$

Step 4: Expressions for triple products in equation (1.2.2) for a_{33}

$$[\vec{n}\ \vec{\omega}^{(F1)}\ \vec{v}^{(F1)}] = E_{11}t_1^2 + E_{12}t_1 + E_{13} \tag{5.6.17}$$

where

$$\begin{aligned} E_{11} &= n_{m1Y}X_{11} + n_{m1X}X_{21} \\ E_{12} &= n_{m1Y}X_{12} - n_{m1X}X_{22} - n_{m1Z}X_{21}\cos\gamma_1 + n_{m1Y}X_{31}\cos\gamma_1 \\ E_{13} &= -(n_{m1Z}X_{22} - n_{m1Y}X_{32})\cos\gamma_1 \end{aligned} \tag{5.6.18}$$

$$[\vec{n}\ \vec{\omega}^{(1)}\ \vec{v}_{tr}^{(F)}] = Y_{21}t_1 + Y_{22} \tag{5.6.19}$$

where

$$\left.\begin{aligned} Y_{21} &= -n_{m1X}X_{21}\sin\gamma_1 + n_{m1Y}(X_{11}\sin\gamma_1 - X_{31}\cos\gamma_1) + n_{m1Z}X_{21}\ cos\gamma_1 \\ Y_{22} &= -n_{m1X}X_{23}\sin\gamma_1 + n_{m1Y}(X_{13}\sin\gamma_1 - X_{33}\cos\gamma_1) \end{aligned}\right\} \tag{5.6.20}$$

$$[\vec{n}\ \vec{\omega}^{(F)}\ \vec{v}_{tr}^{(1)}] = Y_{11}t_1 + Y_{12} \tag{5.6.21}$$

where

$$\left.\begin{aligned} Y_{11} &= -n_{m1Y}(X_{m1}\sin\gamma_1 - Z_{m1}\cos\gamma_1) - n_{m1Y}Y_{m1}\sin\gamma_1 \\ Y_{12} &= \sin\gamma_1 Y_{11} \end{aligned}\right\} \tag{5.6.22}$$

Step 5: Expression for the last term in equation (1.2.2) for a_{33}.

We have to differentiate between two derivatives: $m_{21}^{'}$ and $m_{F1}^{'}$. The first one, $m_{21}^{'}$, is applied to provide a parabolic function of transmissions errors for the case of meshing of the generated

pinion and the gear. Such a function is very useful because it will allow to absorb linear functions of transmission errors caused by the gear misalignment. The other derivative, m'_{F1}, means that the cutting ratio in the process for pinion generation is not constant and it is just an additional parameter of machine-tool settings.

In the approach proposed in this research project it is not required to have modified roll. However the use of such parameter in the more general case with $m'_{F1} \neq 0$ is also included to offer an extra choice. After some derivations, we obtain

$$\frac{(\omega^{(1)})^2}{\omega^{(2)}} m'_{F1}(\vec{n} \cdot \vec{v}_{tr}^{(F)}) = Z_{11}t_1^2 + Z_{12}t_1 + Z_{13} \tag{5.6.23}$$

Here

$$\left.\begin{aligned}
Z_{11} &= (2C)(n_{m1X}X_{11} + n_{m1Y}X_{21} + N_{m1Z}X_{31}) \\
Z_{12} &= (2C)[n_{m1X}X_{13} + n_{m1Y}X_{23} + n_{m1Z}X_{33} + \sin\gamma_1(n_{m1X}X_{11} + N_{m1Y}X_{21} + N_{m1Z}X_{31})] \\
Z_{13} &= (2C)\sin\gamma_1(n_{m1X}X_{13} + n_{m1Y}X_{32} + n_{m1Z}X_{33})
\end{aligned}\right\} \tag{5.6.24}$$

where

$$2C = \frac{m'_{F1}}{(m_{F1})^2} \tag{5.6.25}$$

<u>Step 6</u>: Final expression for a_{33}.

Using the expressions received in steps 4 and 5, we obtain the following expression for a_{33}

$$a_{33} = Z_1t_1^2 + Z_2t + Z_3 \tag{5.6.26}$$

where,

$$Z_1 = \kappa_I^{(F)} L_{21}^2 + \kappa_{II}^{(F)} L_{11} - E_{11} + Z_{11} \tag{5.6.27}$$

$$Z_2 = 2\kappa_I^{(F)} L_{21} L_{22} + 2\kappa_{II}^{(F)} L_{11} L_{12} - E_{12} - Y_{21} + Y_{11} + Z_{12} \tag{5.6.28}$$

$$Z_3 = \kappa_I^{(F)} L_{22}^2 + \kappa_{II}^{(F)} L_{12}^2 - E_{13} - Y_{22} + Y_{12} + Z_{13} \tag{5.6.29}$$

Step 7: New representations of coefficients a_{13} and a_{23}.

Equations (5.6.3), (5.6.7) and (5.6.8) yield

$$\left.\begin{aligned} a_{13} &= N_{21} t_1 + N_{22} \\ a_{23} &= N_{11} t_1 + N_{12} \end{aligned}\right\} \tag{5.6.30}$$

Here;

$$\begin{aligned} N_{11} &= \kappa_{II}^{(F)} L_{11} + M_{21} \\ N_{12} &= \kappa_{II}^{(F)} L_{12} + M_{22} \\ N_{21} &= \kappa_{I}^{(F)} L_{21} + M_{11} \\ N_{22} &= \kappa_{I}^{(F)} L_{22} + M_{12} \end{aligned} \tag{5.6.31}$$

Step 8: Derivation of squared equation for t_1

Equations (5.6.30) and (5.4.5) yield

$$a_1 t_1^2 + a_2 t_1 + a_3 = 0 \tag{5.6.32}$$

where,

$$\left.\begin{aligned} a_1 &= a_{12}Z_1 - N_{21}N_{11} \\ a_2 &= a_{12}Z_2 - (N_{21}N12 + N_{22}N_{11}) \\ a_3 &= a_{12}Z_3 - N_{22}N_{12} \end{aligned}\right\} \tag{5.6.33}$$

Solving equation (5.6.32), we obtain

$$t_1 = \frac{-a_2 \pm \sqrt{a_2^2 - 4a_1a_3}}{2a_1} \tag{5.6.34}$$

There are two solutions for t_1 and we can choose one of them. If the tilt and the modified roll are not used, it can be proven that in this case a_1 becomes equal to zero and equation (5.6.32) yields

$$t_1 = -\frac{a_3}{a_2} \tag{5.6.35}$$

knowing t_1, the ratio of roll may be easily determined as

$$\left.\begin{aligned} m_{F1} &= t_1 + \sin\gamma_1 \\ R_{ap} &= \frac{1}{m_{F1}} \end{aligned}\right\} \tag{5.6.36}$$

According to equations (5.4.19) and (5.6.15), the blank offset and machine center to back can be determined by

$$E_{m1} = \frac{Y_{m1}m_{F1} + X_{11}t_1 + X_{13}}{m_{F1}} \tag{5.6.37}$$

$$X_{G1} = \frac{X_{21}t_1 + X_{23} - x_{m1}m_{F1}}{m_{F1}\cos\gamma_1} \tag{5.6.38}$$

Knowing E_{m1} and X_{G1}, we may represent the position vector of the center of head-cutter with respect to the cradle center as follows,

$$\vec{r}_{m1}^{(O_F)} = \vec{r}_h^{(O_F)} + \begin{bmatrix} X_{G1}\cos\gamma_1 \\ -E_{m1} \\ X_{G1}\sin\gamma_1 + X_{G1} \end{bmatrix} \tag{5.6.39}$$

In practice, the position of the center of the head cutter is defined by radial setting S_{r1} and cradle angle q_1, which may be determined by the following equations,

$$\left.\begin{aligned} S_{r1} &= \sqrt{(X_{m1}^{(O_F)})^2 + (Y_{m1}^{(O_F)})^2} \\ q_1 &= \sin(\frac{-Y_{m1}}{S_{r1}}) \end{aligned}\right\} \tag{5.6.40}$$

Since the cutter center O_F must lie in the machine plane, the component $Z_{m1}^{(O_F)}$ must be zero. Thus, the sliding base X_{B1} may be determined as,

$$X_{B1} = -X_{G1}\sin\gamma_1 \tag{5.6.41}$$

6 Tooth Contact Analysis

6.1 Introduction

The tooth contact analysis (TCA) is directed at simulation of meshing and contact for misaligned gears and enables to determine the influence of errors of manufacturing ,assembly and shaft deflection. The basic equations for TCA are as follows:

$$\vec{r}_h^{(1)}(\theta_F, \phi_F, \phi_1') = \vec{r}_h^{(2)}(\theta_G, \phi_p, \phi_2') \tag{6.1.1}$$

$$\vec{n}_h^{(1)}(\theta_F, \phi_F, \phi_1') = \vec{n}_h^{(2)}(\theta_G, \phi_p, \phi_2') \tag{6.1.2}$$

Equations (6.1.1) and (6.1.2) describe the continuous tangency of pinion and gear tooth surfaces Σ_1 and Σ_2. The subscript h indicates that the vectors are represented in fixed coordinate system S_h. The superscripts 1 and 2 indicate the pinion tooth surface Σ_1 and gear tooth surface Σ_2, respectively. Vector equation (6.1.1) describes that the position vectors of a point on Σ_1 and a point on Σ_2 coincide at the instantaneous point of contact M; vector equation (6.1.2) describes that the surface unit normals coincide at M.

Parameters θ_F and ϕ_F represent the surface coordinates for Σ_1; θ_G and ϕ_p are the surface coordinates for Σ_2. Parameters ϕ_1' and ϕ_2' represent the angles of rotation of the pinion and gear being in mesh.

Two vector equations (6.1.1) and (6.1.2) are equivalent to five independent scaler equations in six unknowns, which are represented as

$$f_i(\theta_F,\ \phi_F,\ \phi_1',\ \theta_G,\ \theta_p,\ \phi_2') = 0 \qquad (i = 1, 2, \cdots, 5) \tag{6.1.3}$$

The continuous solution of equations (6.1.3) means determination of five functions of a parameter chosen as the input one, say ϕ_1'. Such functions are:

$$\theta_F(\phi_1'), \quad \phi_F(\phi_1'), \quad \theta_G(\phi_1'), \quad \phi_p(\phi_1'), \quad \phi_2(\phi_1'), \tag{6.1.4}$$

In accordance with the theorem of Implicit Function System Existence [4], solution (6.1.4) exists if at any iteration the following requirements are observed:

(i) There is a set of parameters

$$P(\theta_F, \quad \phi_F, \quad \theta_G, \quad \phi_p, \quad \phi_2) \tag{6.1.5}$$

that satisfies equations (6.1.1) and (6.1.2)

(ii) The Jacobian that is taken with the above mentioned set of parameters and with ϕ_1' as an independent variable, differs from zero, i.e.

$$\frac{D(f_1, f_2, f_3, f_4, f_5)}{(\theta_F, \phi_F, \theta_G, \phi_p, \phi_2')} \neq 0 \tag{6.1.6}$$

The solution of the system (6.1.3) of nonlinear equations is based on application of a subroutine, such as DNEQNF of the IMSL software package. The first guess f r the starting the iteration process is based on the data that are provided by the local synthesis.

The tooth contact analysis output data, functions (6.1.4), enable to determine the contact path on the tooth surface, the so called line of action, and the transmission errors.

The contact path on pinion tooth surface is determined in S_1 by the following functions

$$\vec{r}_1(\theta_F,\ \phi_F,\ \phi_1') ,\quad \theta_F(\phi_1') ,\quad \phi_F(\phi_1') \tag{6.1.7}$$

Similarly, the contact path on gear tooth surface is represented by functions

$$\vec{r}_2(\theta_G,\ \phi_p,\ \phi_2') ,\quad \theta_G(\phi_1') ,\quad \phi_p(\phi_1') \tag{6.1.8}$$

Function $\phi_2'(\phi_1')$ relates the angles of rotation of the gear and the pinion being in mesh. Deviations of $\phi_2'(\phi_1')$ from the theoretical linear function represent the transmission errors (see section 6.4). TCA is accomplished by the following procedure: (i) derivation of gear tooth surface, (ii) derivation of pinion tooth surface, (iii) determination of transmission errors, and (iv) determination of bearing contact as the set of instantaneous contact ellipses.

6.2 Gear Tooth Surface

The gear tooth surface Σ_2 and the surface unit normal have been represented in S_2 by equations (3.1.5) and (3.2.10), where θ_G is the parameter of generating cone and ϕ_p is the rotational angle of the cradle. Coordinate system S_2 is rigidly connected to the gear. To represent the gear tooth surface Σ_2 and its unit normal in fixed coordinate system S_h we can use the following matrix equations:

$$\vec{r}_h^{(2)}(\theta_G,\ \phi_p,\ \phi_2') = [M_{h2}(\phi_2')]\vec{r}_2(\theta_G, \phi_p) \tag{6.2.1}$$

$$\vec{n}_h^{(2)}(\theta_G,\ \phi_p,\ \phi_2') = [L_{h2}(\phi_2')]\vec{n}_2(\theta_G, \phi_p) \tag{6.2.2}$$

6.3 Pinion Tooth Surface

We will consider two cases for generation of pinion tooth surface: (i) by a cone, and (ii) by a surface of revolution that is formed by rotating curved blades.

Generation by a Cone Surface

Step 1: We recall that the generating cone surface and the surface unit normal has been represented in S_F by equations (5.2.1) and (5.2.3).

$$\vec{r}_F = \begin{bmatrix} (R_{cp} + s_F \sin\alpha_F)\cos\theta_F \\ (R_{cp} + s_F \sin\alpha_F)\sin\theta_F \\ -s_F \cos\alpha_F \\ 1 \end{bmatrix} \tag{6.3.1}$$

$$\vec{n}_F = \begin{bmatrix} -\cos\alpha_F \cos\theta_F \\ -\cos\alpha_F \sin\theta_F \\ -s_F \cos\alpha_F \end{bmatrix} \tag{6.3.2}$$

where S_F and θ_F are the surface coordinates.

Step 2: During the process for generation the cradle with the mounted cone surface performs a rotational motion about the Z_{m1}-axis and a family of cone surfaces with parameter ϕ_F is generated in S_{m1}. This family is represented in S_{m1} by the matrix equation

$$\vec{r}_{m1}(s_F, \theta_F, \phi_F) = [M_{m1c1}(\phi_F)]\vec{r}_{c1}(s_F, \theta_F) \tag{6.3.3}$$

where

$$\vec{r}_{c1} = \vec{r}_F + [\ S_{r1}\cos q_1 \quad -S_{r1}\sin q_1 \quad 0\ ]^T \tag{6.3.4}$$

The position vector $\vec{r}_{c1}$ represents a point of the cone surface in coordinate system S_{c1}; S_{r1} and q_1 are the settings of the head-cutter center O_F in S_{m1}.

Matrix $[M_{m1c1}]$ is (Fig.2.1.1)

$$[M_{m1c1}] = \begin{bmatrix} \cos\phi_F & \sin\phi_F & 0 & 0 \\ -\sin\phi_F & \cos\phi_F & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6.3.5}$$

The unit normal at a point of the generating surface Σ_F is represented in S_{m1} by

$$\vec{n}_{m1}(\theta_F, \phi_F) = [L_{m1c1}(\phi_F)]\vec{n}_{c1}(\theta_F) \tag{6.3.6}$$

where $\vec{n}_{c1} \equiv \vec{n}_F$.

We recall that the generating cone surface is a ruled developed surface and the surface unit normal does not depend on s_F (Parameter s_F determines the location of a point on the cone generatrix.) Matrix $[L_{m1c1}]$ is the 3×3 rotational part of $[L_{m1c1}]$ and is represented as follows,

$$[L_{m1c1}] = \begin{bmatrix} \cos\phi_F & \sin\phi_F & 0 \\ -\sin\phi_F & \cos\phi_F & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{6.3.7}$$

Step 3: Equation of meshing of the head-cutter cone with the pinion tooth surface. The equation of meshing is considered with vectors that are represented in S_{m1}. Thus:

$$\vec{n}_{m1}^{(F)} \cdot \vec{v}_{m1}^{(1F)} = 0 \tag{6.3.8}$$

Here: $\vec{v}^{(F1)}$ is the sliding (relative) velocity represented as follows

$$\vec{v}_{m1}^{(1F)} = (\vec{\omega}_{m1}^{(1)} - \vec{\omega}_{m1}^{(F)}) \times \vec{r}_{m1} + \vec{R}_{m1} \times \vec{\omega}_{m1}^{(1)} \tag{6.3.9}$$

While deriving equation (6.3.9), we have taken into account that vector of angular velocity $\vec{\omega}^{(1)}$ of pinion rotation does not pass through the origin O_{m1} of S_{m1}; $\vec{R}_{m1}$ represents the position vector that is drawn from O_{m1} to a point of line of action of $\vec{\omega}^{(1)}$; $\vec{R}_{m1}$ can be represented as (Fig.2.1.2):

$$\vec{R}_{m1} = [X_{G1}\cos\gamma_1 \quad -E_{m1} \quad X_{G1}\sin\gamma_1]^T \tag{6.3.10}$$

vectors $\vec{\omega}^{(1)}$ and $\vec{\omega}^{(F)}$ are represented in S_{m1} as follows

$$\vec{\omega}_{m1}^{(1)} = [\cos\gamma_1 \quad 0 \quad \sin\gamma_1]^T \qquad (|\vec{\omega}^{(1)} = 1|) \tag{6.3.11}$$

$$\vec{\omega}_{m1}^{(F)} = \frac{1}{R_{ap}}[0 \quad 0 \quad 1]^T \qquad (\frac{1}{R_{ap}} = \frac{\omega^{(F)}}{\omega^{(1)}}) \tag{6.3.12}$$

Equations from (6.3.8) to (6.3.12) yield

$$s_F = \frac{T_1(\theta_F, \phi_F)}{T_2(\theta_F, \phi_F)} \tag{6.3.13}$$

Here:

$$T_1 = X_{nm1}(-E_{m1}\sin\gamma_1 - A_1(\sin\gamma_1 - m_{F1})) + Y_{nm1}(X_{B1}\cos\gamma_1 + A_2(\sin\gamma_1 - m_{F1}))$$
$$+Z_{nm1}(E_{m1}\cos\gamma_1 + A_1\cos\gamma_1) \tag{6.3.14}$$

$$T_2 = X_{nm1}(\sin\gamma_1 - m_{F1})\sin\alpha_F\sin(\theta_F + \phi_F) - Y_{nm1}[(\sin\gamma_1 - m_{F1})\sin\alpha_F\cos(\theta_F + \phi_F)$$
$$-\cos\alpha_F\cos\gamma_1] - Z_{nm1}\cos\gamma_1\sin\alpha_F\sin(\theta_F + \phi_F) \tag{6.3.15}$$

where

$$\left.\begin{aligned} A_1 &= R_{cp}\sin(\theta_F + \phi_F) + S_{r1}\sin(-q_1 + \phi_F) \\ A_2 &= R_{cp}\cos(\theta_F + \phi_F) + S_{r1}\cos(-q_1 + \phi_F) \end{aligned}\right\} \tag{6.3.16}$$

Step 4: Two-parametric representation of surface of action

The surface of action is the set of instantaneous lines of contact between the generating cone surface and the pinion tooth surface that are represented in the fixed coordinate system S_{m1}. The surface of action is represented by equations (6.3.3) and (6.3.13) being considered simultaneously. These equations represent the surface of action by three related parameters. Taking into account equation (6.3.13) , we can eliminate s_F and represent the surface of action in two-parametric form by

$$\vec{r}_{m1} = \vec{r}_{m1}(\theta_F, \phi_F) \tag{6.3.17}$$

The common normal to contacting surfaces has been already represented in two-parametric form by equations (6.3.6).

Generation by a Surface of Revolution

Step 1: The shape of the blades is a circular arc (Fig.5.5.1) and such blades generate a surface of revolution by rotation about the head-cutter axis.

The position-vector of the center of the generating arc is represented in S_{m1} by the equation

$$\vec{\rho}^{(c)}_{m1}(\theta_F, \phi_F) = [M_{m1c1}]\{\vec{\rho}^{(c)} + [\ S_{r1}\cos q_1 \quad -\ S_{r1}\sin q_1 \quad 0]^T\} \tag{6.3.18}$$

where,

$$[M_{m1c1}] = \begin{bmatrix} \cos\phi_F & \sin\phi_F & 0 & 0 \\ -\sin\phi_F & \cos\phi_F & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6.3.19}$$

and $\vec{\rho}^{(c)}$ has been expressed by equation (5.5.7).

Step 2: We will need for further transformations the following equations

$$\vec{e}_{Im_1} = [L_{m1c1}]\vec{e}^{(F)}_{FI} = \begin{bmatrix} -\sin(\theta_F + \phi_F) \\ \cos(\theta_F + \phi_F) \\ 0 \end{bmatrix} \tag{6.3.20}$$

and

$$\vec{\tau}_{m1} = \begin{bmatrix} -\cos(\theta_F + \phi_F) \\ -\sin(\theta_F + \phi_F) \\ 0 \end{bmatrix} \tag{6.3.21}$$

Here: $\vec{e}_{I m_1}^{(F)}$ is the unit vector of principal direction I on the head- cutter surface and $\vec{\tau}_{m1}$ is a unit vector that is perpendicular to $\vec{e}_{I m_1}$ and the axis of the head-cutter (Fig. 6.3.1)

Step 3: To simplify the equation of meshing we will represent it by the following equation.

$$\vec{n}_{m1} \cdot \vec{v}_{m1}^{(1F,C)} = 0 \tag{6.3.22}$$

where $\vec{v}_{m1}^{(1F,C)}$ is the relative velocity of the center of the circular arc that generates the head-cutter surface of revolution. The proof that (6.3.22) is indeed the equation of meshing is based on the following considerations:

(i) The relative velocity for a point of the head-cutter surface is represented by equation (6.3.9), given as

$$\vec{v}_{m1}^{(1F)} = (\vec{\omega}_{m1}^{(1)} - \vec{\omega}_{m1}^{(F)}) \times \vec{r}_{m1} + R_{m1} \times \vec{\omega}_{m1}^{(1)} \tag{6.3.23}$$

We can represent position vector $\vec{r}_{m1}$ for a point M as

$$\vec{r}_{m1} = \vec{r}_{m1}^{(C)} + \rho \vec{n}_{m1} \tag{6.3.24}$$

where ρ is the radius of the arc blade.

While deriving equation (6.3.24), we have taken into account that a normal to the head-cutter surface passes through the current arc center C; the sign of ρ depends on how the surface unit normal is directed with respect to the surface.

Then, we may represent the equation of meshing as follows

$$\begin{aligned}
\vec{v}_{m1}^{(1F)} \cdot \vec{n}_{m1}^{(F)} &= \left\{(\vec{\omega}_{m1}^{(1)} - \vec{\omega}_{m1}^{(F)}) \times [(\vec{r}_{m1} + \rho\vec{n}_{m1}^{(F)}) + (\vec{R}_{m1} \times \vec{\omega}^{(1)})]\right\} \cdot \vec{n}_{m1}^{(F)} \\
&= [(\vec{\omega}_{m1}^{(1)} - \vec{\omega}_{m1}^{(F)}) \times [(\vec{r}_{m1} + (\vec{R}_{m1} \times \vec{\omega}^{(1)})] \cdot \vec{n}_{m1}^{(F)} \\
&= \vec{v}_{m1}^{(1F,C)} \cdot \vec{n}_{m1}^{(F)} = 0 \qquad (6.3.25)
\end{aligned}$$

Thus, equation (6.3.22) is proven.

Step 4: It follows from equation (6.3.22) that vector $\vec{v}_{m1}^{(1F,C)}$ belongs to a plane that is parallel to the tangent plane T to the head- cutter surface (Fig.6.3.2). This means that if vector $\vec{v}_{m1}^{(1F,C)}$ is translated from point C to M it will lie in plane T. The unit vector $\vec{e}_{Im_1}^{(F)}$ lies in plane T already. Then, we may represent the unit normal $\vec{n}_{m1}$ by the equation

$$\vec{n}_{m1}(\theta_F, \phi_F) = \frac{\vec{e}_{Im_1} \times \vec{v}_{m1}^{(1F,C)}}{|\vec{e}_{Im_1} \times \vec{v}_{m1}^{(1F,C)}|} \qquad (6.3.26)$$

where $\vec{v}_{m1}^{(1F,C)}$ is represented as follows,

$$\vec{v}_{m1}^{(F)} = \vec{\omega}^{(F)} \times \vec{\rho}_{m1}^{(C)} \qquad (6.3.27)$$

$$\vec{v}_{m1}^{(1)} = \vec{\omega}^{(1)} \times \left\{\vec{\rho}_{m1}^{(C)} + [-X_{G1}\cos\gamma_1 \quad E_{m1} \quad - X_{G1}\sin\gamma_1]^T\right\} \tag{6.3.28}$$

$$\vec{v}_{m1}^{(1F,C)} = \vec{v}_{m1}^{(1)} - \vec{v}_{m1}^{(F)} \tag{6.3.29}$$

The advantage of vector equation (6.3.26) is that the surface unit normal at the point of contact is represented by a vector function of two parameters only, θ_F and ϕ_F; this vector function does not contain the surface parameter λ.

The order of co-factors in vector equation must provide that the direction of $\vec{n}_{m1}$ is toward the axis of the head-cutter. The direction of $\vec{n}_{m1}$ can be checked with the dot product

$$\triangle = \vec{n}_{m1} \cdot \vec{\tau}_{m1} \tag{6.3.30}$$

The surface unit normal has the desired direction if $\triangle > 0$. In the case when $\triangle < 0$, the desired direction of $\vec{n}_{m1}$ can be observed just by changing the order of co-factors in equation (6.3.26).

To determine parameter λ for the current point of contact we can use the equation,

$$\cos\lambda = \vec{n}_{m1} \cdot \vec{\tau}_{m1} \tag{6.3.31}$$

Step 5: Our final goal is the determination in S_{m1} of a position vector of a current point of contact of surfaces Σ_F and Σ_1. This can be done by using the equation,

$$\vec{r}_{m1}(\theta_F, \phi_F) = \vec{\rho}_{m1}^{(C)} - \rho\vec{n}_{m1} \tag{6.3.32}$$

where ρ is the radius of the circular arc.

Finally, the pinion tooth surface may be determined in S_1 as the set of contact points. Thus:

$$\vec{r}_1(\theta_F, \phi_F) = [M_{1p}][M_{pm_1}]\vec{r}_{m_1}(\theta_F, \phi_F) \tag{6.3.33}$$

The unit normal to surface Σ_1 is determined in S_1 with the equation

$$\vec{n}_1(\theta_F, \phi_F) = [L_{1p}][L_{pm_1}]\vec{n}_{m_1}(\theta_F, \phi_F) \tag{6.3.34}$$

Here: $\vec{r}_{m1}(\theta_F, \phi_F)$ and $\vec{n}_{m1}(\theta_F, \phi_F)$ have been represented by equations (6.3.17) and (6.3.6) for straight blade cutter and by equations (6.3.26) and (6.3.32) for curved blade cutter. Here (Fig.2.1.2):

$$[M_{pm_1}] = \begin{bmatrix} \cos\gamma_1 & 0 & \sin\gamma_1 & -X_{G1}\sin\gamma_1 \\ 0 & 1 & 0 & E_{m_1} \\ -\sin\gamma_1 & 0 & \cos\gamma_1 & -(X_{G1}\sin\gamma_1 + X_{B1}) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6.3.35}$$

$$[M_{1p}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi_1 & \sin\phi_1 & 0 \\ 0 & -\sin\phi_1 & \cos\phi_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6.3.36}$$

where ϕ_1 is the angle of the pinion rotation in the process for generation. Angles ϕ_1 and ϕ_F (the angle of rotation of the cradle) are related as follows:

(i) in the case when the modified roll is not used and R_{ap} is constant, we have

$$\phi_1 = R_{ap}\phi_F \tag{6.3.37}$$

(ii) when the modified roll is used, ϕ_1 is represented by the Taylor's series

$$\phi_1 = f(\phi_F) = R_{ap}(\phi_F - C\phi_F^2 - D\phi_F^3 - E\phi_F^4 - F\phi_F^5) \tag{6.3.38}$$

where C, D, E and F are the coefficients of Taylor's series of generation motion (see Appendix B).

Step 7: The tooth contact analysis, as it was mentioned above, is based on conditions of tangency of the pinion and gear surfaces that are considered in the fixed coordinate system S_h (see section 6.1). To represent the pinion tooth surface and the surface unit normal in S_h we use the matrix equations

$$\vec{r}_h^{(1)} = [M_{h1}]\vec{r}_1^{(1)}(\theta_F, \phi_F) \tag{6.3.39}$$

$$\vec{n}_h^{(1)} = [L_{h1}]\vec{n}_{m_1}^{(1)}(\theta_F, \phi_F) \tag{6.3.40}$$

Here:

$$[M_{h1}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi_1' & -\sin\phi_1' & 0 \\ 0 & \sin\phi_1' & \cos\phi_1' & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6.3.41}$$

where ϕ_1' is the angle of rotation of the pinion being in mesh with the gear.

6.4 Determination of Transmission Errors

The function of transmission errors is determined by the equation

$$\delta(\phi_1') = [\phi_2' - (\phi_2')^0] - \frac{N_1}{N_2}[\phi_1' - (\phi_1')^0] \tag{6.4.1}$$

Here: $(\phi_i')^0 \quad (i = 1, 2)$ is the initial angle of gear rotation with which the contact of surfaces Σ_1 and Σ_2 at the mean contact point is provided. Linear function

$$\frac{N_1}{N_2}[\phi_1' - (\phi_1')^0] \tag{6.4.2}$$

provides the theoretical angle of gear rotation for a gear drive without misalignments. The range of ϕ_2' is determined as follows

$$(\phi_2')^0 - \frac{\pi}{N_2} \leq \phi_2' \leq (\phi_2')^0 + \frac{\pi}{N_2} \tag{6.4.3}$$

The function of transmission errors is usually a piecewise periodic function with period equal to $\phi_i = \frac{2\pi}{N_i}$ $(i = 1,2)$ (Fig.6.4.1). The purpose of synthesis for spiral bevel gears is to provide that the function of transmission errors will be of a parabolic type and of a limited value δ' (Fig.6.4.1).

The tooth contact analysis enables to simulate the influence of errors of assembly of various types, particularly, when the center of the bearing contact is shifted in two orthogonal directions (see section 7).

6.5 Simulation of Contact

Mapping of Contact Path into a Two-Dimensional Space

It was mentioned above that the contact path on the pinion and gear tooth surfaces is determined with functions (6.1.7) and (6.1.8), respectively. For the purpose of visualization , the contact path on the gear tooth surface is mapped onto plane (X_c, Y_c) that is shown in Fig.6.5.1. The X_c-axis is directed along the root cone generatrix and Y_c is perpendicular to the root cone generatrix and passes through the mean contact point (Fig. 6.5.1).

Consider that a current contact point N^* is represented in S_2 (Fig.6.5.2) by coordinates: $X_2(\phi_2'), RL'(\phi_2')$ where ϕ_2' is the angle of rotation of the gear and $RL' = |\overline{EN}| = (Y_2^2 + Z_2^2)^{\frac{1}{2}}$. Axis X_2 belongs to plane (X_c, Y_c) (Fig.6.5.2). While mapping the contact path onto plane (X_c, Y_c), we will represent its current point N^* by N that can be determined by coordinates X_2 and RL', where $RL' = |\overline{EN}| = |\overline{EN^*}|$ (Fig.6.5.3). The coordinates of mean contact point M, XL and R, have been previously determined by equations (3.2.1) and (3.2.2). Drawing of Fig.6.5.3 yield

$$\overline{O_cN} = \overline{O_cO_2} + \overline{O_2E} + \overline{EN} \tag{6.5.1}$$

Here:

$$\overline{O_cO_2} = \overline{O_cK} + \overline{KO_2} \tag{6.5.2}$$

$$\overline{KO_2} = -RL\cos(\gamma_k - \gamma 2)\vec{i}_c \tag{6.5.3}$$

where γ_k is determined by:

$$\gamma_k = \tan^{-1}(\frac{RL}{XL}) \tag{6.5.4}$$

Equations from (6.5.1) to (6.5.8) yield

$$\overline{O_cK} = -|\overline{O_RO_2}|\sin\gamma_2\vec{i}_c \tag{6.5.5}$$

$$\overline{O_2E} = X_2\cos\gamma_2\vec{i}_c - X_2\sin\gamma_2\vec{j}_c \tag{6.5.6}$$

$$\overline{EN} = RL'(\sin\gamma_2\vec{i}_c + \cos\gamma_2\vec{j}_c) \tag{6.5.7}$$

$$\overline{O_cN} = X_c\vec{i}_c + Y_c\vec{j}_c \tag{6.5.8}$$

$$\left.\begin{aligned} X_c &= X_2(\phi_2')\cos\gamma_2 + RL'(\phi_2')\sin\gamma_2 - [(XL)^2 + (RL)^2]^{\frac{1}{2}}\cos(\gamma_k - \gamma_2) \\ Y_c &= X_2(\phi_2')\sin\gamma_2 + RL'(\phi_2')\cos\gamma_2 - Z_R\sin\gamma_2 \end{aligned}\right\} \tag{6.5.9}$$

Contact Ellipse

Theoretically, the tooth surfaces of the pinion and the gear are in point contact. However, due to the elastic deformation of tooth surfaces their contact will be spread over an elliptic area. The dimensions and orientation of the instantaneous contact ellipse depend on the elastic approach δ of the surfaces and the principal curvatures and the angel $\sigma^{(12)}$ formed between principal directions $\vec{e}_I^{(1)}$ and $\vec{e}_I^{(2)}$ of the surfaces. The elastic approach depends on the magnitude of the applied load. The value of δ can be taken from experimental results and this will enable us to consider the determination of the instantaneous contact ellipse as a geometric problem. Usually, the magnitude δ is taken as $\delta = 0.00025$ inch.

In our approach the curvatures and principal directions of the pinion and the gear are determined with the principal curvatures and directions of the generating tools and parameters of relative motion in the process for generation.

Gear Tooth Principal Curvatures and Directions

The procedure for determination of gear tooth principal curvatures and directions was described in section 1.2. Knowing functions $\theta_p(\phi_2'), \phi_p(\phi_2')$ from the TCA procedure of computation, we are able to determine the position vector $\vec{r}_{m2}(\theta_p(\phi_2'), \phi_p(\phi_2'))$ and the surface unit normal $\vec{n}_{m2}(\theta_p(\phi_2'), \phi_p(\phi_2'))$ for an instantaneous point of contact. The principal directions and curvatures for the generating surface can be determined from equations (5.2.4), (5.2.5) and (5.2.6). The parameters of relative motions in the process for generation can be determined with equations (3.1.12) and (3.1.13).

Pinion Tooth Principal Curvatures and Directions

As it was mentioned above, the pinion tooth surface can be generated by a cone or by a surface of revolution. The derivation of principal curvatures and directions on the pinion tooth surface is based on relations between principal curvatures and directions between mutually enveloping surfaces Σ_F of the head- cutter and Σ_1 of the pinion. The procedure of derivation is as follows:

Step 1: We represent in S_{m1} the principal directions on the head- cutter surface Σ_F using the following equations

$$\vec{e}^{(F)}_{m_1 j} = [L_{m_1 c_1}]\vec{e}^{(F)}_{jc_1} \qquad (j = I, II) \tag{6.5.10}$$

Step 2: Parameters of relative motion in the process for pinion generation have been represented by equations (5.4.14) to (5.4.19). The derivative of cutting ratio, m'_{F1}, is equal to zero for the case when the modified roll is not used, and can be determined when the modified roll is applied as follows (see the Appendix)

$$m'_{F1} = \frac{d^2\phi_F}{d\phi_1^2} = -\frac{f''(\phi_F)}{[f'(\phi_F)]^3} \tag{6.5.11}$$

where,

$$f'(\phi_F) = R_{ap}(1 - 2C\phi_F - 3D\phi_F^2 - 4E\phi_F^3 - 5F\phi_F^4)$$

$$f''(\phi_F) = -R_{ap}(2C + 6D\phi_F + 12E\phi_F^2 + 20F\phi_F^3)$$

(6.5.12)

Step 3: Now, since the principal curvatures and directions on Σ_F are known and the relative motion is also known, we can determine for each point of contact path the principal curvatures κ_I and κ_{II} of the pinion tooth surface Σ_1, the angle $\sigma^{(F1)}$ and the principal directions $\vec{e}^{(1)}_{Im_1}$, $\vec{e}^{(1)}_{IIm_1}$ on Σ_1. We use for this purpose equations (1.2.6) to (1.2.10). The principal directions on Σ_1 can be represented in coordinate system S_h by the matrix equation (Fig.5.2.2),

$$\vec{e}_{hj}^{(1)} = [L_{h1}][L_{1p}][L_{pm_1}]\vec{e}_{m_1 j}^{(1)} \qquad (j = I, II) \tag{6.5.13}$$

Orientation and Dimensions of the Instantaneous Contact Ellipse

Knowing the principal directions and principal curvatures for the contacting surfaces at each point of contact path, we can determine the half-axes a and b of the contact ellipse and angle $\alpha^{(1)}$ of the ellipse orientation (Fig.6.5.4). The procedure of computation is as follows [4]:

Step 1: Determination of a and b

$$A = \frac{1}{4}\left[K_{\Sigma}^{(1)} - K_{\Sigma}^{(2)} - \sqrt{g_1^2 - 2g_1g_2\cos 2\sigma + g_2^2}\right] \tag{6.5.14}$$

$$B = \frac{1}{4}\left[K_{\Sigma}^{(1)} - K_{\Sigma}^{(2)} + \sqrt{g_1^2 - 2g_1g_2\cos 2\sigma + g_2^2}\right] \tag{6.5.15}$$

$$a = \sqrt{\left|\frac{\delta}{A}\right|} \tag{6.5.16}$$

$$b = \sqrt{\left|\frac{\delta}{B}\right|} \tag{6.5.17}$$

where,

$$K_{\Sigma}^{(i)} = K_{I}^{(i)} + K_{II}^{(ii)} \qquad g_i = K_{I}^{(i)} - K_{II}^{(ii)} \qquad (i = 1, 2) \tag{6.5.18}$$

Step 2: Determination of $\sigma^{(12)}$ (Fig.6.5.4)

$$\left.\begin{aligned} \sin\sigma^{(12)} &= [\ \vec{n}_h\ \vec{e}_{Ih}^{(1)}\ \vec{e}_{Ih}^{(2)}\ ] \\ \cos\sigma^{(12)} &= \vec{e}_{Ih}^{(1)} \cdot \vec{e}_{Ih}^{(2)} \end{aligned}\right\} \tag{6.5.19}$$

Step 3: Determination of $\alpha^{(1)}$

Angle $\alpha^{(1)}$ determines the orientation of the long axis of the contact ellipse with respect to $\vec{e}_{hI}^{(1)}$ (Fig.6.5.4) and is one of the angles determined by the following equations,

$$\tan 2\alpha^{(1)} = \frac{g_2 \sin 2\sigma^{(12)}}{g_1 - g_2 \cos 2\sigma^{(12)}} \tag{6.5.20}$$

Step 4: The orientation of unit vectors $\vec{\eta}$ and $\vec{\zeta}$ of long and short axes of the contact ellipse (Fig.6.5.4) with respect to the pinion principal directions is determined with the equations

$$\vec{\eta}_h = \vec{e}_{hI}^{(1)} \cos\alpha^{(1)} - \vec{e}_{hII}^{(1)} \sin\alpha^{(1)} \tag{6.5.21}$$

$$\vec{\zeta}_h = \vec{e}_{hI}^{(1)} \sin\alpha^{(1)} + \vec{e}_{hII}^{(1)} \cos\alpha^{(1)} \tag{6.5.22}$$

Step 5: In order to visualize the contact ellipse we represent its axes of contact ellipse in plane (X_c, Y_c) (Fig.6.5.1), using the following equations

$$\vec{\eta}_2 = [L_{2h}]\vec{\eta}_h \qquad \vec{\zeta}_2 = [L_{2h}]\vec{\zeta}_h \tag{6.5.23}$$

where

$$[L_{2h}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\cos\phi_2' & -\sin\phi_2' \\ 0 & \sin\phi_2' & -\cos\phi_2' \end{bmatrix} \begin{bmatrix} \cos\Gamma & 0 & -\sin\Gamma \\ 0 & 1 & 0 \\ \sin\Gamma & 0 & \cos\Gamma \end{bmatrix} \tag{6.5.24}$$

Axes of the contact ellipse form with the gear axes the following angles

$$\left.\begin{aligned} \triangle_\eta &= \arccos(\vec{\eta}_2 \cdot \vec{i}_2) \\ \triangle_\zeta &= \arccos(\vec{\zeta}_2 \cdot \vec{i}_2) \end{aligned}\right\} \tag{6.5.25}$$

The unit vectors of axes of contact ellipse form in plane (X_c, Y_c) the following angles with the X_c-axis (the generatrix of the root cone) :

$$\tau_1 = \triangle_\eta - \gamma_2 \qquad \tau_2 = \triangle_\zeta - \gamma_2 \tag{6.5.26}$$

7 V and H check

The purpose of the so called V and H check is the computer aided simulation of the shift of the bearing contact to the toe and to the hill of the gear. The gear quality is judged with the sensitivity of the shape of the contact pattern and the change in the level of transmission errors to the above-mentioned shift of contact.

7.1 Determination of V and H values

Fig.7.1.1 shows the initial position M of contact point (it is the mean contact point), and the new position M^* of the contact point). The shift of the contact pattern was caused by the deformation under the load. Coordinates XL and RL determines the location of M. For the following derivations we will use the following notations.

(i) $PF = A - A^*$ *is the shift of the center of bearing contact*, where F is the tooth length measuring along the pitch line; p is an algebraic value, that is positive when $A^* < A$ and the shift is performed to the toe as shown in Fig.7.1.1. Usually, p is equal to 0.25.

(ii) δ_G and α_G are the gear dedendum and addendum angle.

(iii) $PD = b_G$ and $P^*D^* = b_G^*$ are the gear dedendums that are measured in sections I and I^*.

(iv) $h_m = BD$ and $h^* = B^*D^*$ are the gear tooth heights.

(v) Γ_2 is the pitch cone angle

The determination of V and H for point contact M^* is based on the following procedure.

Step 1: Determination of XL^* and RL^*.

Fig.7.1.1 results in :

$$h^* = h_m - pF(\tan\delta_G + \tan\alpha_G) \tag{7.1.1}$$

$$b_G^* = b_G - pF\tan\delta_G \tag{7.1.2}$$

where $b_G^* = P^*D^*$ and $b_G = PD$

We assume that $M^*D^* = \frac{h^* + c}{2}$ and $MD = \frac{h+c}{2}$, where c is the clearance.

Taking into account that

$$\overline{O_2M^*} = \overline{O_2P^*} + \overline{P^*M^*} \tag{7.1.3}$$

we obtain

$$XL^* = A^*\cos\Gamma_2 + P^*M^*\sin\Gamma_2 = A^*\cos\Gamma_2 + (b_G^* - \frac{h^* + C}{2})\sin\Gamma_2 \tag{7.1.4}$$

$$RL^* = A^*\sin\Gamma_2 + P^*M^*\cos\Gamma_2 = A^*\sin\Gamma_2 - (b_G^* - \frac{h^* + C}{2})\cos\Gamma_2 \tag{7.1.5}$$

The surface coordinates (θ_G^*, ϕ_p^*) can be determined by solving the following two equations,

$$X_2(\theta_G^*, \phi_p^*) = XL^* \tag{7.1.6}$$

$$[Y_2(\theta_G^*, \phi_p^*)]^2 + [Z_2(\theta_G^*, \phi_p^*)]^2 = (RL^*)^2 \tag{7.1.7}$$

Step 2: Determination of V and H

We introduce the shift of the bearing contact in coordinate system S_h by V and H that are directed along the shortest distance between the pinion and gear axes, and the pinion axis, respectively (Fig.7.1.2). V is positive when the gear is shifted apart from the pinion in Y_h direction, H is positive when the pinion is withdrawn. It is obvious that

$$[\vec{r}_h^{(2)}]^* = \vec{r}_h^{(2)} + V\vec{j}_h \tag{7.1.8}$$

$$[\vec{r}_h^{(1)}]^* = \vec{r}_h^{(1)} + H\vec{j}_h \tag{7.1.9}$$

Here: $\vec{r}_h^{(i)}$ $(i = 1,2)$ is the position vector for the initial point of contact, $[\vec{r}_h^{(i)}]^*$ $(i = 1,2)$ is the position vector for the shifted contact point; $\vec{i}_h, \vec{j}_h$ and $\vec{k}_h$ are the unit vectors of coordinate axes S_h.

Equations of tangency at the new contact point provide

$$[\vec{r}_h^{(2)}(\theta_G^*, \phi_p^*, \phi_2')]^* = [\vec{r}_h^{(1)}(\theta_F^*, \phi_F^*, \phi_1')]^* \tag{7.1.10}$$

$$[\vec{n}_h^{(2)}(\theta_G^*, \phi_p^*, \phi_2')]^* = [\vec{n}_h^{(1)}(\theta_F^*, \phi_F^*, \phi_1')]^* \tag{7.1.11}$$

Gear surface coordinates θ_G^* and ϕ_p^* can be determined from equations (7.1.6) and (7.1.7). Equations (7.1.10) and (7.1.11) yield

$$V = [Y_h^{(2)}(\theta_G^*, \phi_p^*, \phi_2')]^* \quad [Y_h^{(1)}(\theta_F^*, \phi_F^*, \phi_1')]^* \tag{7.1.12}$$

$$H = [X_n^{(2)}(\theta_G^*, \phi_p^*, \phi_2^{'})]^* - [X_h^{(1)}(\theta_F^*, \phi_F^*, \phi_1^{'})]^* \tag{7.1.13}$$

$$[Z_h^{(2)}(\theta_G^*, \phi_p^*, \phi_2^{'})]^* - [Z_h^{(1)}(\theta_F^*, \phi_F^*, \phi_1^{'})]^* = 0 \tag{7.1.14}$$

$$\left.\begin{aligned} \sin\phi_1^{'} &= \frac{-n_{hY}^{(2)}n_{1Z}^{(1)} + n_{hZ}^{(2)}n_{1Y}^{(1)}}{(n_{1Y}^{(1)})^2 + (n_{1Z}^{(1)})^2} \\ \cos\phi_1^{'} &= \frac{n_{hY}^{(2)}n_{1Y}^{(1)} + n_{hZ}^{(2)}n_{1Z}^{(1)}}{(n_{1Y}^{(1)})^2 + (n_{1Z}^{(1)})^2} \end{aligned}\right\} \tag{7.1.15}$$

$$n_{hX}^{(2)}(\theta_G^*, \phi_p^*, \phi_2^{'})]^* - n_{hX}^{(1)}(\theta_F^*, \phi_F^*, \phi_1^{'})]^* = 0 \tag{7.1.16}$$

Equations from (7.1.12) to (7.1.16) represent a system of five independent equations in six unknowns: V, H, $\phi_2', \phi_1', \theta_F$ and ϕ_F. The sixth independent equation, that is required for the solution of unknowns, can be derived based on the condition that the equation of meshing must be satisfied with the designed gear ratio, i.e.,

$$\vec{n}_h^{(2)} \cdot \vec{v}_h^{(12)} = f(\theta_G^*, \phi_G^*, \phi_2^{'}, \phi_1^{'}, \theta_F, \phi_p, V, H) = 0 \tag{7.1.17}$$

In solving the above system, we first solve a sub-system composed of equations (7.1.14), (7.1.16) and (7.1.17) for ϕ_2', ϕ_F and θ_F, and then calculate the values of ϕ_1', V and H directly, by equations (7.1.12), (7.1.13) and (7.1.15).

7.2 Tooth Contact Analysis for Gears with Shifted Center of Bearing Contact

After the determination of parameters V and H, the tooth contact analysis for gears with shifted center of bearing contact can be performed similarly to the analysis described in sections 6.4. and 6.5. The initial guess for the first iteration in the procedure of computations is provided by the set of six unknowns obtained in section 7.1.

Appendix A

Generation with Modified Roll

1 Introduction

Modification of roll or sometimes called modified roll means that the cutting ratio is not constant but varied in the process for generation. The variable cutting ratio–the variable ratio of roll– can be provided by a cam mechanism of the transmission of the cutting machine or by the servo-motors of a computer controlled cutting machine. According to the developments of Gleason, the TCA program can analyze the process for generation up to members of the fifth order. However, due to the limitations caused by application of cam mechanisms only the parameters up to the third order are controllable in the process for generation.

The modified roll is an additional parameter for the synthesis of spiral bevel gears. In our approach the synthesis of spiral bevel gears can be performed, as it was mentioned above, with a constant cutting ratio. However, we consider in this section the application of modified roll as well to provide a broader point of view on synthesis of spiral bevel gears.

2 Taylor Series for the Function of Generation Motion

According to the practice of Gleason, the kinematic relation between the angles of rotation of the workpiece and the cradle is represented by a Taylor's series up to fifth order. To the knowledge of the authors, Gleason has never published any materials related to the kinematics of the modified

roll. However, Professor Zheng had done a good job in deciphering Gleason's mechanisms for modified roll and represented the kinematic relations in his valuable book [5].

Consider that the angles of rotation of the pinion and the cradle are related by a nonlinear function

$$\phi_1 = f(\phi_F) \qquad f \in C^K \quad (K \geq 3) \tag{A.1}$$

We assume that $\phi_1 = 0$ at $\phi_F = 0$ and represent $f(\phi_F)$ in the neighborhood of $\phi_F = 0$ by the Taylor series as follows,

$$\phi_1 = f'(0)\phi_F + \frac{1}{2!}f''(0)\phi_F^2 + \cdots \tag{A.2}$$

Taking into account that

$$\frac{d\phi_1}{d\phi_F} = f'(\phi_F) \tag{A.3}$$

We obtain

$$f'(0) = \frac{\omega^{(1)}}{\omega^{(F)}}|_{\phi_F=0} = R_{ap} \tag{A.4}$$

where R_{ap} is the ratio of roll.

Without loosing generality of the solution, we can take $\omega^{(1)} = 1$ and then obtain

$$f^{'}(\phi_F)\frac{d\phi_F}{dt} = 1 \tag{A.5}$$

Differentiation of equation (A.5) yields

$$f^{''}(\phi_F)(\frac{d\phi_F}{dt})^2 = f^{'}(\phi_F)\frac{d^2\phi_F}{dt^2} \tag{A.6}$$

Equation (A.6) yields

$$\frac{a_2}{\omega_F^2} = -\frac{f^{''}(\phi_F)}{f^{'}(\phi_F)} \tag{A.7}$$

where $a_2 = \frac{d^2\phi_F}{d^2t^2}$ is the angular acceleration of the cradle.

Equation (A.7) with new designations can be represented as follows

$$2C = \frac{a_2}{\omega_F^2} = -\frac{1}{R_{ap}}f^{''}(0) \tag{A.8}$$

Similar differentiation of higher order of equation (A.3) yields:

$$\phi_1 = R_{ap}(\phi_F - C\phi_F^2 - D\phi_F^3 - E\phi_F^4 - F\phi_F^5) \tag{A.9}$$

Here:

$$2C = -\frac{1}{R_{ap}} f''(0)$$

$$6D = -\frac{1}{R_{ap}} f'''(0)$$

$$24E = -\frac{1}{R_{ap}} f^{IV}(0)$$

$$120F = -\frac{1}{R_{ap}} f^{V}(0)$$

Unfortunately, function $f(\phi_F)$ cannot be represented in explicit form for certain cutting machines, for instance, for the Gleason spiral bevel grinder. For such a case we will consider the following auxiliary expressions

$$a_3 = \frac{d^3\phi_F}{dt^3} \quad , \quad 6CX = \frac{a_3}{\omega_F^3} \tag{A.10}$$

$$a_4 = \frac{d^4\phi_F}{dt^4} \quad , \quad 24DX = \frac{a_4}{\omega_F^4} \tag{A.11}$$

$$a_5 = \frac{d^5\phi_F}{dt^5} \quad , \quad 120EX = \frac{a_5}{\omega_F^5} \tag{A.12}$$

Then, differentiating equation (A.6) and taking $\phi_F = 0$, we may obtain the following equations

$$6D = 6CX - 3(2C)^2 \tag{A.13}$$

$$24E = 24DX + (2C)[15(2C)^2 - 10(6CX)] \tag{A.14}$$

$$120F = 120EX - 15(2C)(24DX) + 105(2C)^2[6CX - (2C)^2] - 10(6CX)^2 \tag{A.15}$$

The procedure for determination of coefficients C, D, E and F for the Taylor's series (A.9) when function $f_1(\phi_F)$ cannot be represented in explicit form is as follows:

Step 1: Differentiate the implicit equation that relates ϕ_F and ϕ_1 up to five times and then find $\omega_F, a_2, a_3, a_4, a_5$ in terms of ϕ_1 and ϕ_2 at $\phi_1 = \phi_F = 0$.

Step 2: Considering $\phi_1 = \phi_F = 0$, find $6CX, 24DX, 120EX$ by equations (A.10) - (A.12).

Step 3: Find $2C, 6D, 24E, 120F$ by equation (A.8),(A.13)- (A.15).

3 Synthesis of Gleason's Cam

Introduction

Gleason's cam mechanism,as shown schematically in Fig. A.3.1, is an ingenious invention that has been proposed and developed by the engineers of the Gleason Works. The mechanism transforms rotation of the cam about O_q into rotation of the cradle about O_c. The rotation of the cam about O_q is related with the rotation of the pinion being generated, but the angles of cam rotation and pinion rotation, ϕ_q and ϕ_1, are related by a linear function when there is no cam settings.

To authors' knowledge, the engineers of the Gleason Works have not published the principles of synthesis and analysis of this mechanism. However, H.Cheng [6], Zheng [5] have made good contributions to the deciphering of this mechanism. The following is a systematic representation of synthesis and analysis of Gleason's mechanism.

The purpose of cam synthesis is to obtain the shape of the cam, considering that the angles of rotation of the cam and the cradle are related by a linear function, $\phi_c(\phi_2)$. However, this function

can be modified into a nonlinear function by changing the location of the designed cam with respect to O_q and the orientation of the cam guides that are installed on the cradle. Fig. A.4.1 shows the settings of the cam mechanism with the designed shape : (i) the cam is translated along the line O_cO_q an amount $\triangle T$; (ii) and then, the cam guides are rotated about the cam rotation center and formed angle α with O_cO_q. It is obvious that the cam mechanism with the settings $\triangle T$ and α will transform rotation about O_q to O_c with a nonlinear function between the angles of rotation of the cam and the cradle. The deviation of this function from a linear one depends on settings of the cam mechanism and will be discussed in section A.4.

Coordinate Systems

While considering the synthesis of the cam mechanism, we will use three coordinate systems: the movable coordinate systems S_c and S_q that are rigidly connected to the cradle and the cam, and S_f that is the fixed coordinate system (Fig. A.3.2).

Equation of Meshing, Contact Point in S_c

Assuming that the transformation of motion is performed with constant ratio of angular velocities and in the same direction, we can determine the location of instantaneous center of rotation, I, in coordinate system S_f by using the equation (Fig. A.3.2)

$$\frac{\omega_q}{\omega_c} = \frac{E + r_u}{r_u} \qquad \text{(A.16)}$$

Where, E in the distance between the cradle center O_c and the cam rotation center O_q; r_u is the so-called pitch radius of the cam.

The location of instantaneous point of contact M on the guides can be determined by using the theorem of planar gearing [4]. According to this theorem the common normal to the guides and the cam at the point of their contact must pass through the instantaneous center of rotation I. Thus, contact point M and the unit normal at M are represented in S_c as follows

$$r_c = [\ -b\ \ -u\ \ \ 0\ \ \ 1\ \ ]^T \tag{A.17}$$

$$n_c = [\ 1\ \ \ 0\ \ \ 0\ \ ]^T \tag{A.18}$$

(A.19)

Here: b is an algebraic value (b is positive if the left side of guides is considered and b is negative if the right side of guides is considered); u is a variable parameter that is determined with the equation

$$u = (E + r_u)\cos\theta_c - E \tag{A.20}$$

Equations (A.17) and (A.20) yield

$$\vec{r}_c(\theta_c) = [\ -b\ \ f(\theta_c)\ \ 0\ \ 1\ ]^T \tag{A.21}$$

where

$$f(\theta_c) = E - (E + r_u)\cos\theta_c \tag{A.22}$$

Shape of the Cam

The shape of the cam is a planar curve that is represented in S_q by the matrix equation

$$\vec{r}_q(\theta_c) = [M_{qp}][M_{pf}][M_{fc}]\vec{r}_c(\theta_c) \tag{A.23}$$

Here: coordinate system S_p is an auxiliary fixed coordinate system (Fig. A.3.2). Matrices in equation (A.23) are represented as follows

$$[M_{qp}] = \begin{bmatrix} \cos\theta_q & -\sin\theta_q & 0 & 0 \\ \sin\theta_q & \cos\theta_q & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.24}$$

$$[M_{pf}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & E \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.25}$$

$$[M_{fc}] = \begin{bmatrix} \cos\theta_c & \sin\theta_c & 0 & 0 \\ -\sin\theta_c & \cos\theta_c & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.26}$$

The normal to the cam shape is represented by the matrix equation

$$\vec{n}_q(\theta_c) = [L_{qp}][L_{pf}][L_{fc}]\vec{n}_c(\theta_c) \tag{A.27}$$

Here: $[L_{pf}]$ is the identity matrix and is the (3×3) submatrix of the respective matrix $[M]$. We consider that the shape of the cam and its normal depend on the generalized parameter θ_q only since

$$\theta_c = (\frac{r_u}{E + r_u})\theta_q \tag{A.28}$$

The final equations of the cam and its normal are represented as follows

$$\vec{r}_q = \begin{bmatrix} -b\cos(\frac{E}{E+r_u}\theta_q) + (r_u + E)\cos(\frac{r_u}{r_u+E}\theta_q)\sin(\frac{E}{E+r_u}\theta_q) - E\sin\theta_q \\ -b\sin(\frac{E}{E+r_u}\theta_q) - (r_u + E)\cos(\frac{r_u}{r_u+E}\theta_q)\cos(\frac{E}{E+r_u}\theta_q) - E\cos\theta_q \\ 0 \\ 1 \end{bmatrix} \tag{A.29}$$

$$\vec{n}_q = \begin{bmatrix} \cos(\frac{E}{E+r_u}\theta_q) \\ \sin(\frac{E}{E+r_u}\theta_q) \\ 0 \end{bmatrix} \tag{A.30}$$

4 Cam Analysis

The cam analysis is directed at the determination of function $\theta_c^*(\theta_q^*)$ for a cam and guides with modified settings. The analysis is based on simulation of tangency of the designed cam with the cradle guides taking into account the settings of the cam and the guides.

Coordinate Systems and Coordinate Transformation

Coordinate systems S_d, S_r and S_c are rigidly connected to the guides and the cradle (Fig. A.4.1(a)). The guides after rotation about O_c form angle α with the y_c-axis.

Coordinate systems S_q, S_n and S_m are rigidly connected to the cam. The settings of the cam with respect to S_m are determined by $\triangle T$ and angle α.

The cam and the cradle perform rotations about O_m and O_f, respectively (Fig. A.4.2). The conditions of continuous tangency mean that the designed cam and the guides have a common normal and a common position vector at every instant in S_f.

A current point N of the guide is determined in S_f with the equation (Fig. A.4.1 and Fig. A.4.4):

$$\vec{r}_f^{(1)} = [M_{fc}^*][M_{ce}][M_{ed}]\vec{r}_d \tag{A.31}$$

where

$$\vec{r}_d = [\ -b\ \ -\lambda\ \ \ 0\ \ \ 1\ ]^T \tag{A.32}$$

The unit normal is determined in S_f as follows

$$\vec{n}_f^{(1)} = [L_{fc}^*][L_{ce}][L_{ed}]\vec{n}_d \tag{A.33}$$

where

$$\vec{n}_d = [\ 1\ \ \ 0\ \ \ 0\ ]^T \tag{A.34}$$

Here:

$$[M^*_{fc}] = \begin{bmatrix} \cos\theta^*_c & -\sin\theta^*_c & 0 & 0 \\ \sin\theta^*_c & \cos\theta^*_c & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.35}$$

$$[M_{ce}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -E \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.36}$$

$$[M_{ed}] = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 \\ -\sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.37}$$

A current point of the cam and the unit normal at this point are represented in S_f by the equations (Fig. A.4.1(b), Fig. A.4.2).

$$\vec{r}_f^{(2)} = [M^*_{fp}][M^*_{pm}][M^*_{mn}][M^*_{nq}]\vec{r}_q \tag{A.38}$$

$$\vec{n}_f^{(2)} = [L^*_{fp}][L^*_{pm}][L^*_{mn}][L^*_{nq}]\vec{n}_q \tag{A.39}$$

Equations (A.38) and (A.23) yield

$$\vec{r}_f^{(2)} = [M_{fp}^*][M_{pm}^*][M_{mn}^*][M_{nq}^*][M_{qp}][M_{pf}][M_{fc}]\vec{r}_c \tag{A.40}$$

where

$$\vec{r}_c = [\; -b \;\; -u \;\; 0 \;\; 1 \;]^T \tag{A.41}$$

Matrices $[M_{qp}], [M_{pf}]$ and $[M_{fc}]$ have been represented by equations (A.24), (A.25) and (A.26), respectively. Matrices $[M_{fp}^*], [M_{pm}^*], [M_{mn}^*]$ and $[M_{nq}^*]$ are represented as follows (Fig. A.4.1(b), Fig. A.4.2):

$$[M_{fp}^*] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -E \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.42}$$

$$[M_{pm}^*] = \begin{bmatrix} \cos\theta_q^* & -\sin\theta_q^* & 0 & 0 \\ \sin\theta_q^* & \cos\theta_q^* & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.43}$$

$$[M_{ed}] = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 \\ -\sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.44}$$

$$[M^*_{nq}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\triangle T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.45}$$

Equations (A.39) and (A.27) yield

$$\vec{n}_f^{(2)} = [L^*_{fp}][L^*_{pm}][L^*_{mn}][L^*_{nq}][L_{qp}][L_{pf}][L_{fc}]\vec{n}_c \tag{A.46}$$

Here:

$$[\ \vec{n}_c\ ] = [\ 1 \quad 0 \quad 0\ ] \tag{A.47}$$

Matrices $[L^*]$ and $[L]$ are 3×3 submatrices of matrices $[M^*]$ and $[M]$.

Equations of Tangency

The tangency of cam and guides with modified settings is represented by equations

$$\vec{r}_f^{(1)}(\theta_c^*, \theta_c, \alpha) = \vec{r}_f^{(2)}(\theta_q^*, \theta_q, \triangle T, \alpha) \tag{A.48}$$

$$\vec{n}_f^{(1)}(\theta_c^*, \theta_c, \alpha) = \vec{n}_f^{(2)}(\theta_q^*, \theta_q, , \alpha) \tag{A.49}$$

We recall that vector equations (A.48) and (A.49) yield a system of only three independent equations in four unknowns: $\theta_c^*, \theta_q^*, \theta_c$ and λ; setting parameters $\triangle T$ and α are considered as given; θ_q and θ_c are related with equation (A.28) and θ_q is considered as a generalized parameter. Our goal is to determine the function that relates angles of rotation of the cam and the cradle, ϕ_q^* and ϕ_c^*, and the parameters of settings α and $\triangle T$, i.e. the function

$$F(\theta_q^*, \theta_c^*, \triangle T, \alpha) = 0 \tag{A.50}$$

Equality of Contact Normal–Satisfaction of Equation (A.49)

Equations (A.49), (A.33) and (A.46) yield

$$[L_{fc}^*][A] = [L_{pm}^*][A][L_{qp}][L_{fc}] \tag{A.51}$$

Then we obtain

$$[B][A] = [A][c] \tag{A.52}$$

Here:

$$[B] = [L^*_{fc}][L^*_{pm}] = \begin{bmatrix} \cos(\theta^*_q - \theta^*_c) & \sin(\theta^*_q - \theta^*_c) & 0 \\ -\sin(\theta^*_q - \theta^*_c) & \cos(\theta^*_q - \theta^*_c) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A.53}$$

$$[C] = [L_{qp}][L_{fc}] = \begin{bmatrix} \cos(\theta_q - \theta_c) & -\sin(\theta_q - \theta_c) & 0 \\ \sin(\theta_q - \theta_c) & \cos(\theta_q - \theta_c) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A.54}$$

$$[A] = [L_{cd}] = [L^*_{nq}] = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A.55}$$

Matrices (A.52) are rotational matrices that describe rotation about axes of the same orientation. This means that we can change the order of co-factor matrices and

$$[B][A] = [C][A] \tag{A.56}$$

This yields that

$$[C]^{-1}[B][A] = [A] \tag{A.57}$$

$$[C]^{-1}[B] = [I] \tag{A.58}$$

where $[I]$ is a unitary matrix and

$$[B] = [C] \tag{A.59}$$

Equation (A.59) yields

$$\theta_q^* - \theta_c^* = -(\theta_q - \theta_c) = -\frac{E}{E + r_u}\theta_q \tag{A.60}$$

since

$$\theta_c = \frac{E}{E + r_u}\theta_q \tag{A.61}$$

Equality of Position Vectors–Satisfaction of Equation (A.48)

Equations (A.48), (A.31) and (A.38) yield

$$[M_{pm}^*]^{-1}[M_{fp}^*]^{-1}[M_{fc}^*][M_{cc}][M_{ed}]\vec{r}_d = [M_{mn}^*][M_{nq}^*][M_{qp}][M_{pf}][M_{fc}]\vec{r}_c \tag{A.62}$$

After transformations we obtain

$$[Q]\vec{r}_d = [S]\vec{r}_c \tag{A.63}$$

Here:

$$[Q] = \begin{bmatrix} a_{11} & a_{12} & 0 & a_{14} \\ a_{21} & a_{22} & 0 & a_{24} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.64}$$

$$[S] = \begin{bmatrix} a_{11} & a_{12} & 0 & a^{*}_{14} \\ a_{21} & a_{22} & 0 & a^{*}_{24} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{A.65}$$

The rotational 3×3 submatrices of $[Q]$ and $[S]$ are equal due to the equality of contact normals (see equation (A.52). The elements of $[Q]$ and $[S]$ are represented by

$$a_{11} = \cos\eta \quad a_{12} = \sin\eta \quad a_{21} = -a_{12} \quad a_{22} = a_{11} \tag{A.66}$$

where $\eta = \theta^{*}_{q} - \theta^{*}_{c} + \alpha$, and

$$a_{14} = E[-\sin(\theta^{*}_{q} - \theta^{*}_{c}) + \sin\theta^{*}_{q}] \quad , \quad a_{24} = E[-\cos(\theta^{*}_{q} - \theta^{*}_{c}) + \cos\theta_{q}] \tag{A.67}$$

$$a^{*}_{14} = -E[\sin(\theta_{q} - \alpha) - \triangle T \sin\alpha] \quad , \quad a^{*}_{24} = E[\cos(\theta_{q} - \alpha) - \triangle T \cos\alpha] \tag{A.68}$$

Matrix equation (A.62) yields the following system of two linear equations

$$a_{12}(u - \lambda) + a_{14} - a^{*}_{14} = 0 \quad , \quad a_{22}(u - \lambda) + a_{24} - a^{*}_{24} = 0 \tag{A.69}$$

Eliminating $(u - \lambda)$, we obtain

$$(a_{14} - a_{14}^{*})a_{22} - (a_{24} - a_{24}^{*})a_{11} = 0 \tag{A.70}$$

Equations (A.70), (A.66), (A.67) and (A.68) results in

$$F(\theta_q^*, \theta_c^*, \triangle T, \alpha) = \sin[\frac{r_u}{E}(\theta_q^* - \theta_c^*)] + \frac{\triangle T}{E}\sin(\theta_q^* - \theta_c^*) - \sin(\theta_c^* - \alpha) - \sin\alpha = 0 \tag{A.71}$$

Equation (A.71) represents in implicit form the displacement function for the cam mechanism with settings α and $\triangle T$. It is easy to be verified, that equation (A.71) with $\triangle T = 0, \alpha = 0$ represents the linear function ,

$$\theta_q = \frac{E + r_u}{r_u}\theta_c \tag{A.72}$$

For Gleason's grinder, E is equal to 15 inch. According to Gleason's practice, the sense of rotation of the cradle and the cam is opposite to the assumption in the derivation in this report. Without loss of generality, by substituting $\theta_q^* = -\theta_q^*$ and $\theta_c^* = -\theta_c^*$ with $E = 15$ in equation (A.71) we obtain the final expression of the relation between θ_c^* and θ_q^* as follows,

$$\sin(\theta_c^* + \alpha) - \sin\alpha + \frac{\triangle T}{15}\sin(\theta_c^* - \theta_q^*) + \sin\frac{r_u}{15}(\theta_c^* - \theta_q^*) = 0 \tag{A.73}$$

5 Determination of Coefficients of the Taylor's Series

The determination of the coefficients of the Taylor's Series for generation motion with modified roll is a lengthy process. Gleason provides its customers with computer program which can select the cams with settings and analyze the effects of the modified roll. However Gleason's program is a black box with no explanation for the determination of the coefficients of the Taylor's Series. Valuable contribution to the understanding of Gleason's program has been made by C.Q. Zheng [5]. For reader's convenience, a series of derivations are represented in this section, which coincide with the equations in [5] except some printing errors.

In the process of generation, the cam rotates at a constant angular velocity . Without loss of generality, we assume that the cam rotates with unitary velocity, i.e. $\frac{d\theta_q^*}{dt} = 1$. Using the procedure discussed in section A.2, we differentiate equation (A.73) five time as follows,

$$\omega_c^* \cos(\theta_c^* + \alpha) + (\omega_c^* - 1)[\frac{\triangle T}{15}\cos(\theta_c^* - \theta_q^*) + \frac{r_u}{15}\cos\frac{r_u}{15}\sin(\theta_c^* - \theta_q^*)] = 0 \qquad \text{(A.74)}$$

$$\begin{aligned} &a_2 \cos(\theta_c^* + \alpha) + (\omega_c^*)^2 \sin(\theta_c^* + \alpha) \\ &+a_2[\frac{r_u}{15}\cos\frac{r_u}{15}(\theta_c^* - \theta_q^*) + \frac{\triangle T}{15}\cos(\theta_c^* - \theta_q^*)] \\ &-(\omega_c^* - 1)^2[(\frac{r_u}{15})^2 \sin\frac{r_u}{15}(\theta_c^* - \theta_q^*)] + \frac{\triangle T}{15}\sin(\theta_c^* - \theta_q^*)] = 0 \end{aligned} \qquad \text{(A.75)}$$

$$\begin{aligned} &a_3 \cos(\theta_c^* + \alpha) - 3a_2(\omega_c^*)^2 \sin(\theta_c^* + \alpha) - (\omega_c^*)\cos(\theta_c^* + \alpha) \\ &+a_3[\frac{r_u}{15}\cos\frac{r_u}{15}(\theta_c^* - \theta_q^*) + \frac{\triangle T}{15}\cos(\theta_c^* - \theta_q^*)] \\ &-3a_2(\omega_c^* - 1)[(\frac{r_u}{15})^2 \sin\frac{r_u}{15}(\theta_c^* - \theta_q^*) + \frac{\triangle T}{15}\sin(\theta_c^* - \theta_q^*)] \\ &-(\omega_c^* - 1)^3[(\frac{r_u}{15})^3 \cos\frac{r_u}{15}(\theta_c^* - \theta_q^*) + \frac{\triangle T}{15}\cos(\theta_c^* - \theta_q^*)] = 0 \end{aligned} \qquad \text{(A.76)}$$

$$
\begin{aligned}
& a_4 \cos(\theta_c^* + \alpha) - 4a_3\omega_c^* \sin(\theta_c^* + \alpha) - 3a_2^2 \sin(\theta_c^* + \alpha) \\
& -6(\omega_c^*)^2 a_2 \cos(\theta_c^* + \alpha) + (\omega_c^*)^4 \sin(\theta_c^* + \alpha) \\
& -4a_3(\omega_c^* - 1)[(\frac{r_u}{15})^2 \sin \frac{r_u}{15}(\theta_c^* - \theta_q^*) + \frac{\triangle T}{15} \cos(\theta_c^* - \theta_q^*)] \\
& -3a_2^2[(\frac{r_u}{15})^2 \sin \frac{r_u}{15}(\theta_c^* - \theta_q^*) + \frac{\triangle T}{15} \sin(\theta_c^* - \theta_q^*)] \\
& +a_4[\frac{r_u}{15} \cos \frac{r_u}{15}(\theta_c^* - \theta_q^*) + \frac{\triangle T}{15} \cos(\theta_c^* - \theta_q^*)] \\
& -6a_2(\omega_c^* - 1)^2[(\frac{r_u}{15})^3 \cos \frac{r_u}{15}(\theta_c^* - \theta_q^*) + \frac{\triangle T}{15} \cos(\theta_c^* - \theta_q^*)] \\
& (\omega_c^* - 1)^4[(\frac{r_u}{15})^4 \sin \frac{r_u}{15}(\theta_c^* - \theta_q^*) + \frac{\triangle T}{15} \sin(\theta_c^* - \theta_q^*)] = 0
\end{aligned}
\tag{A.77}
$$

$$
\begin{aligned}
& a_5 \cos(\theta_c^* + \alpha) - [10a_3(\omega_c^*)^2 + 15a_2^2\omega_c^* - (\omega_c^*)^5] \cos(\theta_c^* + \alpha) \\
& -[5a_4\omega_c^* + 10a_2a_3 - 10a_2(\omega_c^*)^3] \sin(\theta_c^* + \alpha) \\
& +a_5[\frac{r_u}{15} \cos \frac{r_u}{15}(\theta_c^* - \theta_q^*) + \frac{\triangle T}{15} \cos(\theta_c^* - \theta_q^*)] \\
& -5a_4(\omega_c^* - 1)[(\frac{r_u}{15})^2 \sin \frac{r_u}{15}(\theta_c^* - \theta_q^*) + \frac{\triangle T}{15} \sin(\theta_c^* - \theta_q^*)] \\
& -10a_3(\omega_c^* - 1)^2[(\frac{r_u}{15})^3 \cos \frac{r_u}{15}(\theta_c^* - \theta_q^*) + \frac{\triangle T}{15} \cos(\theta_c^* - \theta_q^*)] \\
& -10a_2a_3[(\frac{r_u}{15})^2 \sin \frac{r_u}{15}(\theta_c^* - \theta_q^*) + \frac{\triangle T}{15} \sin(\theta_c^* - \theta_q^*)] \\
& -15a_2^2(\omega_c^* - 1)[(\frac{r_u}{15})^3 \cos \frac{r_u}{15}(\theta_c^* - \theta_q^*) + \frac{\triangle T}{15} \cos(\theta_c^* - \theta_q^*)] \\
& +10a_2(\omega_c^* - 1)^3[(\frac{r_u}{15})^4 \sin \frac{r_u}{15}(\theta_c^* - \theta_q^*) + \frac{\triangle T}{15} \sin(\theta_c^* - \theta_q^*)] \\
& +(\omega_c^* - 1)^5[(\frac{r_u}{15})^5 \cos \frac{r_u}{15}(\theta_c^* - \theta_q^*) + \frac{\triangle T}{15} \cos(\theta_c^* - \theta_q^*)] = 0
\end{aligned}
\tag{A.78}
$$

At $\theta_c^* = \theta_q^* = 0$, we can determine $\omega_c^*, a_2, a_3, a_4, a_5$ from above expressions as follows,

$$
\omega_c^* = \frac{r_u + \triangle T}{15 \cos \alpha + r_u + \triangle T} \tag{A.79}
$$

$$a_2 = \frac{15\sin\alpha}{15\cos\alpha + r_u + \triangle T}(\omega_c^*)^2 \tag{A.80}$$

$$a_3 = \frac{3a_2\omega_c^*\sin\alpha + (\omega_c^*)^3\cos\alpha + (\omega_c^* - 1)^3(\frac{r_u^3}{15^3} + \frac{\triangle T}{a5})}{\cos\alpha + \frac{r_u + \triangle T}{15}} \tag{A.81}$$

$$a_4 = \frac{6a_2(\omega_c^*)^2\cos\alpha + [4a_3\omega_c^* + 3a_2^2 - (\omega_c^*)^4]\sin\alpha + 6a_2(\omega_c^* - 1)^2(\frac{r_u^3}{15^3} + \frac{\triangle T}{a5})}{\cos\alpha + \frac{r_u + \triangle T}{15}} \tag{A.82}$$

$$\begin{aligned} a_5 = & \frac{1}{\cos\alpha + \frac{r_u + \triangle T}{15}}\{[10a_3(\omega_c^*)^2 + 15a_2^2\omega_c^* - (\omega_c^*)^5]\cos\alpha \\ & +[5a_4\omega_c^* + 10a_2a_3 - 10a_2(\omega_c^*)^3]\sin\alpha + [10a_3(\omega_c^* - 1)^3 \\ & +15a_2^2(\omega_c^* - 1)](\frac{r_u^3}{15^3} + \frac{\triangle T}{a5}) - (\omega_c^* - 1)^3(\frac{r_u^3}{15^3} + \frac{\triangle T}{a5})\} \end{aligned} \tag{A.83}$$

Using equation (A.8) and (A.10) - (A.12), we obtain

$$R_{ac} = \frac{1}{\omega_c^*} = 1 + \frac{15\cos\alpha}{r_u + \triangle T} \tag{A.84}$$

$$2C = \frac{R_{ac} - 1}{R_{ac}}\tan\alpha \tag{A.85}$$

$$6CX = \frac{1 + 3(2C)\tan\alpha + \frac{(1 - R_{ac})^3}{15\cos\alpha}(\frac{r_u^3}{15^3} + \triangle T)}{1 + \frac{r_u + \triangle T}{15\cos\alpha}} \tag{A.86}$$

$$\begin{aligned} 24DX = & \frac{1}{\cos\alpha + \frac{r_u + \triangle T}{15}}\{6(2C)\cos\alpha + [4(6CX) + 3(2C)^2 - 1]\sin\alpha \\ & +6(2C)(1 - R_{ac})^2(\frac{r_u^3}{15^3} + \frac{\triangle T}{15})\} \end{aligned} \tag{A.87}$$

$$
\begin{aligned}
120EX &= \frac{1}{\cos\alpha + \frac{r_u + \triangle T}{15}}\{[10(6CX) + 15(2C)^2 - 1]\cos\alpha \\
&\quad +[5(24DX) + 10(2C)(6CX) - 10(2C)]\sin\alpha \\
&\quad +[10(6CX)(1 - R_{ac})^2 + 15(2C)^2(1 - R_{ac}](\frac{r_u^3}{15^3} + \frac{\triangle T}{15}) \\
&\quad -(1 - R_{ac})^5(\frac{r_u^5}{15^5} + \frac{\triangle T}{15})\}
\end{aligned} \tag{A.88}
$$

Knowing $R_{ac}, 2C, 6CX, 24DX$ and $120EX$, we can determine $6D, 24E$ and $120F$ by equations (A.13) - (A.15).

6 Selection of Cams and Cam Settings

In order to provide the desired low transmission errors and bearing contact, the ratio of roll R_{ap} and second ratio of roll ($2c$), which are determined by the local synthesis, must be applied for the grinder. Due to the structure of Gleason's grinder, the ratio of roll, R_{ap} is related to R_{ac} as follows,

$$
R_{ac} = m_i m_c R_{ap} \tag{A.89}
$$

Here, R_{ac} is the transmission ratio between the cam and the cradle, as determined by equation (A.84). m_c is a fixed gear ratio and is equal to 1 in Gleason's grinder; m_i is the gear ratio from the workpiece to the cam and is determined as,

$$
m_i = \frac{n}{n_i} \tag{A.90}
$$

here, n is the number of teeth of the workpiece and n_i is the index internal, i.e. the gear tooth number skipped over in indexing.

From equation (A.80) and (A.89), we obtain

$$\alpha = \tan^{-1}[(\frac{(2c)R_{ac}}{R_{ac}-1})] \tag{A.91}$$

$$r_u + \triangle T = \frac{15\cos\alpha}{R_{ac}-1} \tag{A.92}$$

The cams and their pitch radii r_u are tabulized. A cam with pitch radius closest to $(r_u + \triangle T)$ calculated by equation (A.92) should be selected. After the cam with pitch radius r_u is selected the corresponding setting, $\triangle T$, can then be determined as:

$$\triangle T = \frac{15\cos\alpha}{R_{ac}-1} - r_u \tag{A.93}$$

In some cases, it is also necessary to control $6CX$. In order to satisfy $R_{ac}, 2C$ and $6CX$, the value of n_i can be used together with $\triangle T$ and α. Since n_i must be an integral number it is difficult to obtain an accurate solution. But by careful selection of cams and index interval n_i, a practical engineering solution is often achievable.

When the cam and its settings are selected, it is then necessary to determine the coefficients of the Taylor's Series of the generation motion and carry out the TCA to see how the higher order coefficients (i.e., $6D, 24E$ and $120F$) affect the transmission errors and bearing contact. If the result of TCA are satisfactory, then the gears can be ground by the selected cam and cam settings.

Appendix B

Description of Program and Numerical Example

Input and Output of Program

The research project is complemented by a computer program, which can be used for the determination of machine tool settings through the method of local synthesis and simulate the transmission errors and bearing contact through TCA. The input data to the program include four parts.

Part 1. Blank Data

TN1 : pinion number of teeth

TN2 : gear number of teeth

C : shaft offset (zero for spiral bevel gear)

FW : width of gear

GAMMA : shaft angle

MCD : mean Cone distance

RGMA1 : pinion root cone angle

B1 : pinion spiral angle

B2 : gear spiral angle

RGMA2 : gear root cone angle

FGMA2 : gear face cone angle

PGMA2 : gear pitch cone angle

D2R : gear root cone apex beyond pitch apex

D2F : gear face cone apex beyond pitch apex

ADD2 : gear mean addendum

DED2 : gear mean dedendum

WD : whole depth

CC : clearance

DEL : elastic approach (experiment datum)

Part 2. Cutter Specifications

RU2 : gear nominal cutter radius

PW2 : point width of gear cutter

ALP2 : blade angle of gear cutter

Part 3. Parameters of Synthesis Condition

FI21 : derivative of transmission ratio, negative for gear convex side and positive for gear concave side. The range is $-0.008 \leq FI21 \leq 0.008$.

KD : percentage of the half long axes of contact ellipse over face width. $KD = 0.15 - 0.20$.

ETAG : direction angle of contact path. For right hand gear, $-80^\circ \leq ETAG \leq 0^\circ$ for gear convex side and $-80^\circ \leq ETAG \leq 0^\circ$ for gear concave side; For left hand gear, $0^\circ \leq ETAG \leq 80^\circ$ for gear convex side and $-80^\circ \leq ETAG \leq 0$ for gear concave side. When ETAG is close to zero, the contact path is along the tooth height, when the magnitude is increased, the contact path will have bias in and reach almost longitudinal direction if ETAG is close to 90 degrees.

GAMA1 : pinion machine root angle, which is the same as the pinion root angle if no tilt is used.

RHO : radius of the arc blade if curved blade is used, which can be any values when curved blade is not used.

C2 : second order ratio of roll if modified roll is used. C2 is zero without modified roll.

ALP1 : pinion cutter blade angle. ALP1 is positive for gear convex side and negative for gear concave side. ALP1 can be the same as ALP2. For better result, it is suggested for pinion concave side the magnitude of ALP1 is smaller than ALP2 and for pinion convex side, the magnitude of ALP1 is larger than ALP2.(As shown in the example).

TN1I : number of teeth skipped over indexing. TN1I is only used in modified roll, the ratio between TN1I and TN1 must not be an integer.

Part 2. Control Codes

JCL : JCL control V and H check, $JCL = 1$ means no V-H check.

JCH : For right hand gear, set $JCH = 1$, for left hand gear set $JCH = 2$.

JCC : For straight blade, set $JCC = 1$, for curved blade set $JCC = 2$.

TL1, TL2 : Extra points on contact path, both should be less or equal than 2.

The program output includes: (1) the machine-tool settings for gear and pinion; (2) the transmission error; (3) the contact path; (4) the length and orientation of the long axes of the contact ellipse; and bearing contact at toe and heel position.

Numerical Example

The model used in this report is the spiral bevel drive with the shaft angle of 90 degrees. In the numerical example, modified roll and curved blade for generation of gears were not used since favorable results were attained without them. The list of the blank data and machine tool settings are tabulized in the attached tables.

The TCA results with V-H check are shown through Fig. B.1 through Fig. B.6. The V and H values shown in the figures are of $\frac{1}{1000}$ inch. It is shown also that the transmission errors are very small and the bearing contact is stable for both side at the three positions, toe, mean and heel.

BLANK DATA

	PINION	GEAR
NUMBER OF TEETH:	11	41
PRESSURE ANGLE:	20°	
SHAFT ANGLE:	90°	
MEAN SPIRAL ANGLE:	35.0°	
HAND OF SPIRAL:	LF	RH
OUTER CONE DISTANCE:		90.07
FACE WIDTH:		27.03
WHOLE DEPTH:	8.11	8.11
CLEARANCE:	0.81	0.81
ADDENDUM:	5.24	2.061
DEDENDUM:	2.87	6.05
PITCH ANGLE:	15°1'	74°59'
ROOT ANGLE:	13°20'	70°39'
FACE ANGLE:	19°21'	76°40'

GEAR CUTTER SPECIFICATIONS

BLADE ANGLE:	20°
CUTTER DIAMETER:	152.40
POINT WIDTH:	2.79

GEAR MACHINE TOOL SETTINGS

RADIAL SETTING(s):	70.43577
CRADLE ANGLE(q):	62.3981°
MACHINE CENTER TO BACK(X_G):	0.00
SLIDING BASE(X_B):	0.00
RATIO OF ROLL(R_a):	1.032397
BLANK OFFSET(E_m):	0.0
MACHINE ROOT ANGLE(γ_m):	70.65°

PINION MACHINE TOOL SETTINGS

	CONVEX	CONCAVE
CUTTER BLADE ANGLE:	21.5°	18.5°
CUTTER POINT RADIUS:	80.4876	71.7222
RADIAL SETTING(s):	71.55166	69.04316
CRADLE ANGLE(q):	59.4638°	64.0624°
MACHINE CENTER TO BACK(X_G):	1.08497	-1.58960
SLIDING BASE(X_B):	-0.25021	0.36659
RATIO OF ROLL(R_a):	3.898097	3.788604
BLANK OFFSET(E_m):	-2.56862(Up)	2.19033(Down)
MACHINE ROOT ANGLE(γ_m):	13.3333°	13.3333°

References

[1] Gleason Works: Understanding Tooth Contact Analysis. Rochester, NY 14692, Publication No. SD3139, 1981.

[2] Litvin, F.L. and Gutman, Y.: Methods of Synthesis and Analysis for Hypoid Gear Drives of "Formate" and "Helixform", Parts 1-3. ASME Journal of Mechanical Design. Vol. 103, January 1981, pp.83-113.

[3] Litvin, F.L. et al: Topology of Modified Helical Gears. Surface Topology, Vol.2,Issue 1, March 1989, pp.41-59.

[4] Litvin, F. L.: Theory of Gearing, NASA publication: 1212 (AVSCOM technical report; 88-c-035),1989.

[5] Chang-Qi Zheng: Spiral Bevel and Hypoid Gear Drives, in Chinese, Press of Mechanical Industry, 1988, Beijing.

[6] H.C. Chao and H.S. Cheng: A Computer Solution for the Dynamic Load, Lubricant Film Thickness and Surface Temperatures in Spiral Bevel Gears. NASA Contract Report, 4077, July, 1987.

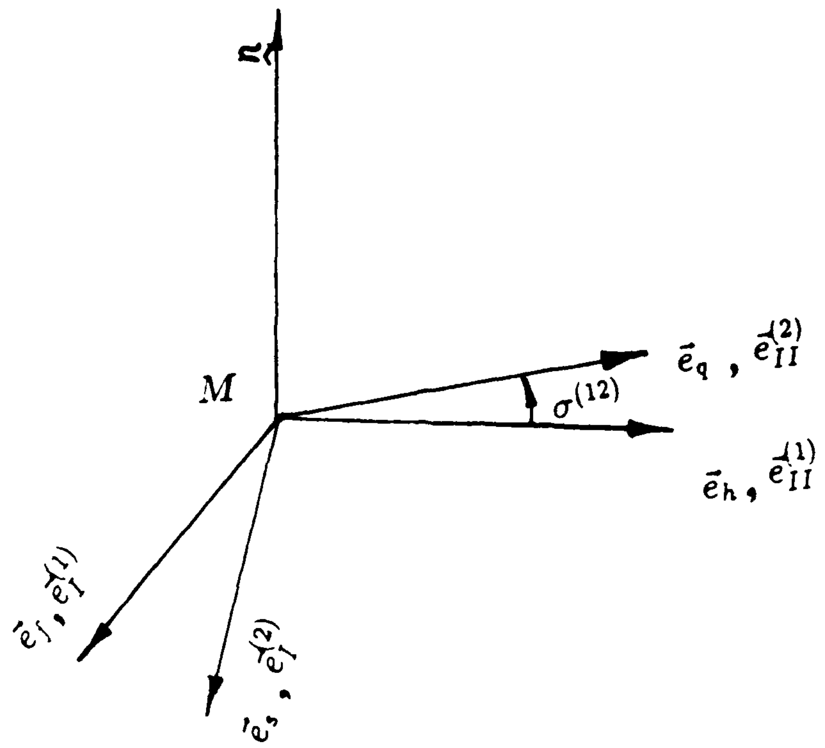


Fig. 1.2.1 Unit Vectors of Principal Directions

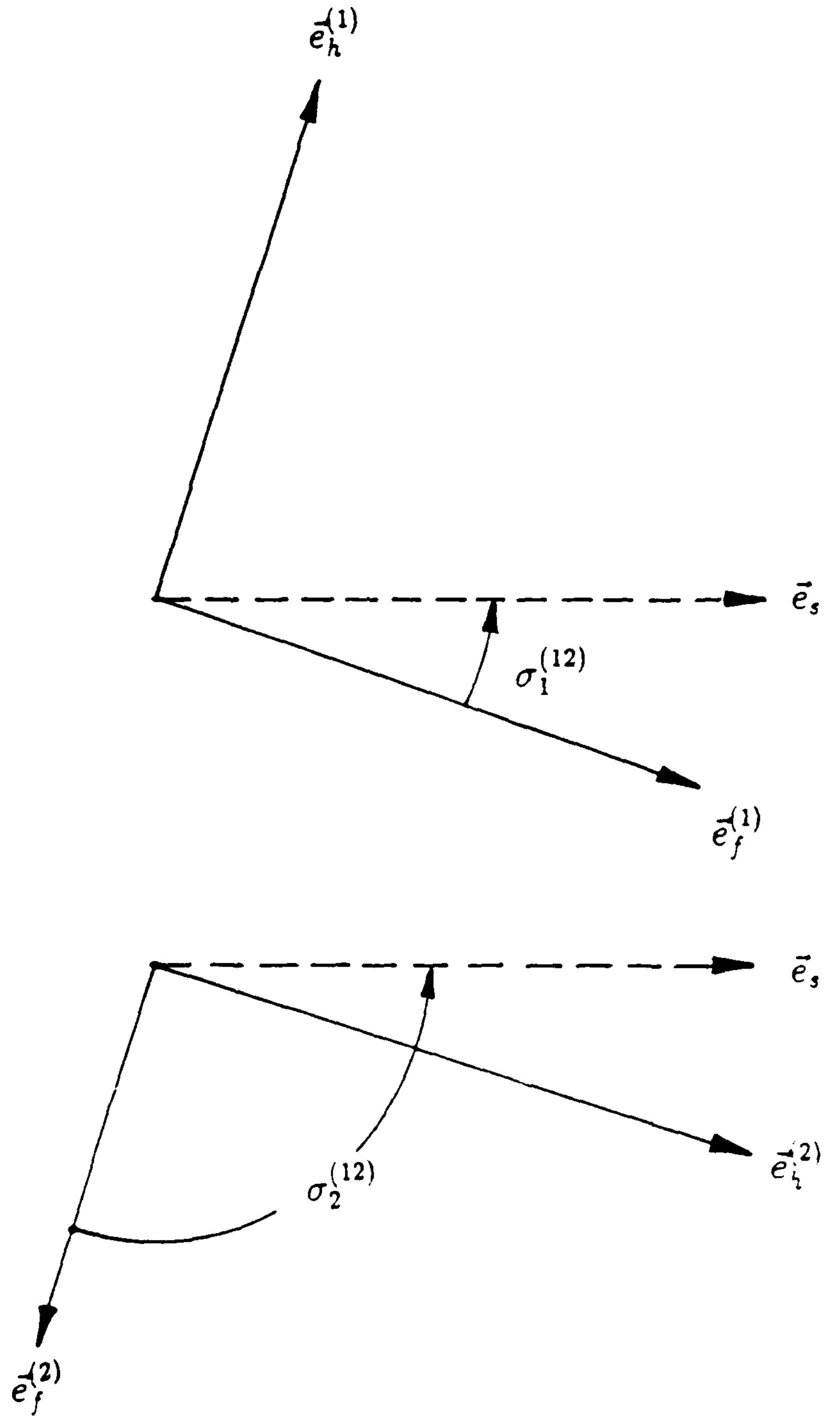


Fig. 1.2.2 The two Solutions for $\sigma^{(12)}$

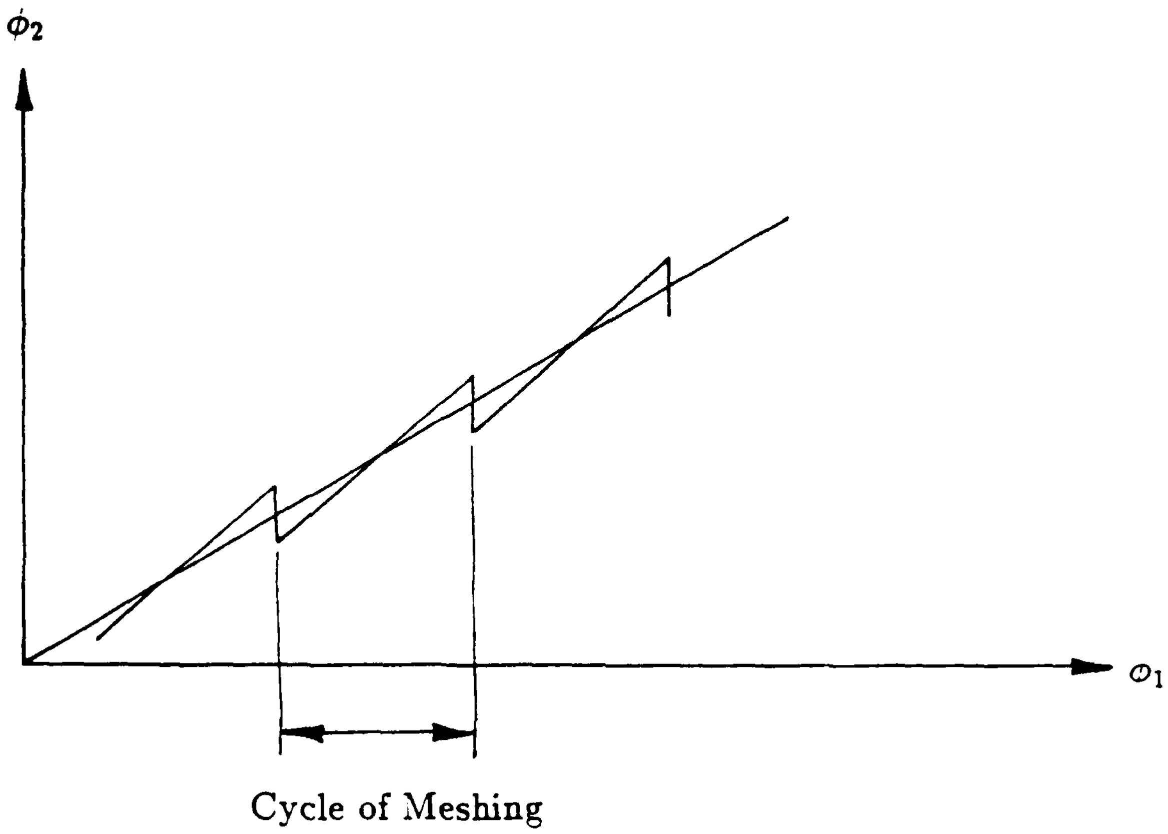


Fig. 1.2.3 Piecewise Linear Function of Transmission Errors

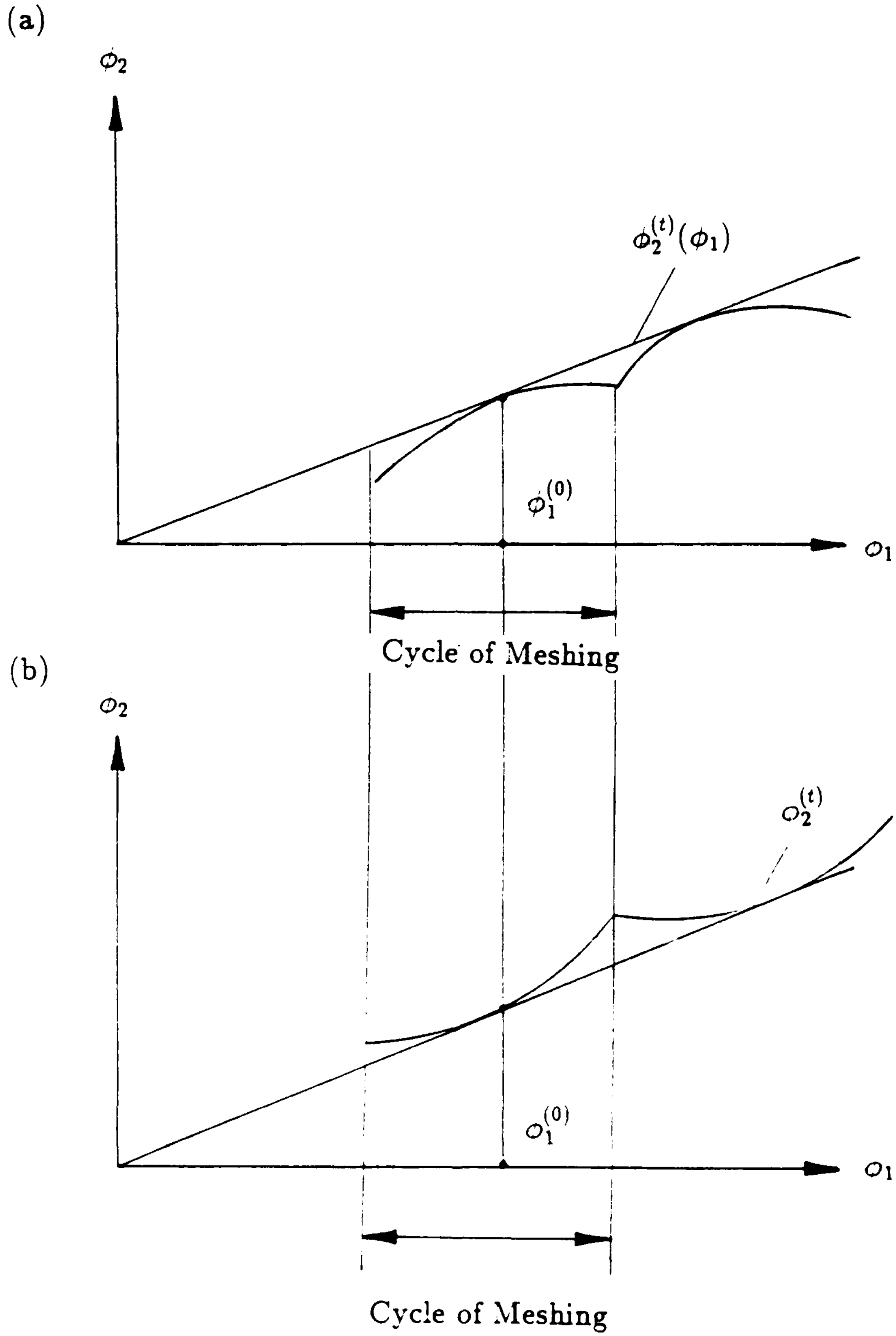


Fig. 1.2.4 Parabolic Type of Transmission Errors

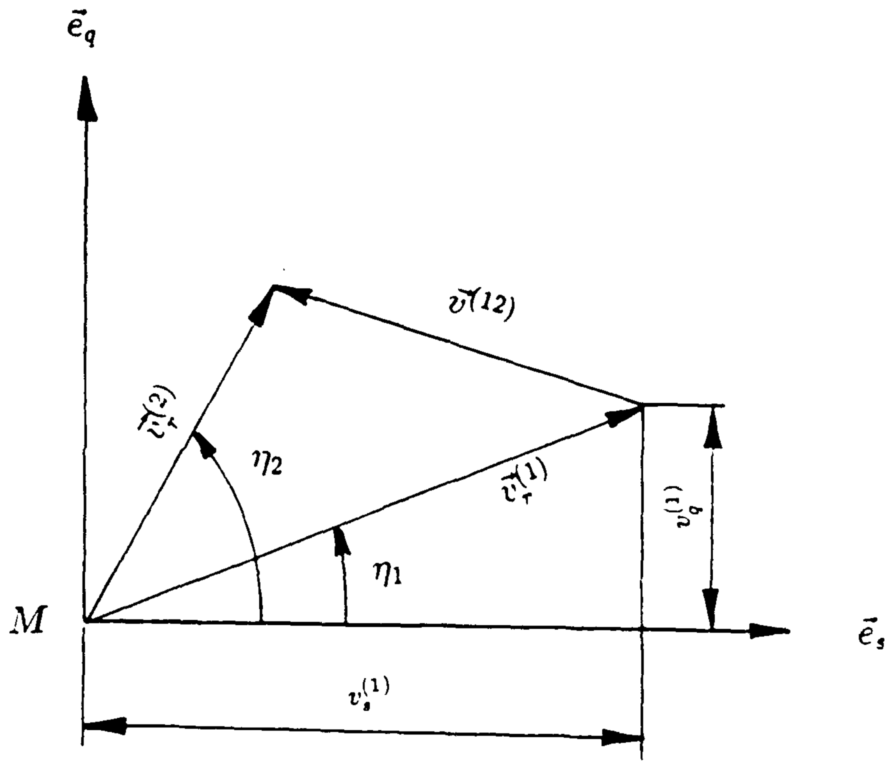


Fig. 1.2.5 Tangents of Contact Paths

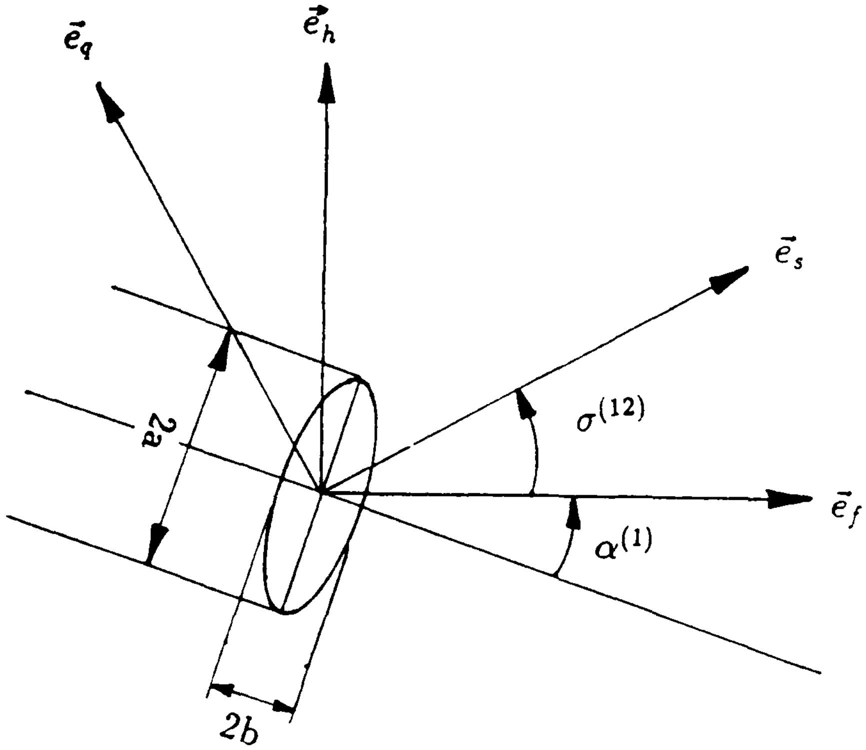


Fig. 1.2.6 Orientation and Dimension of Contact Ellipse

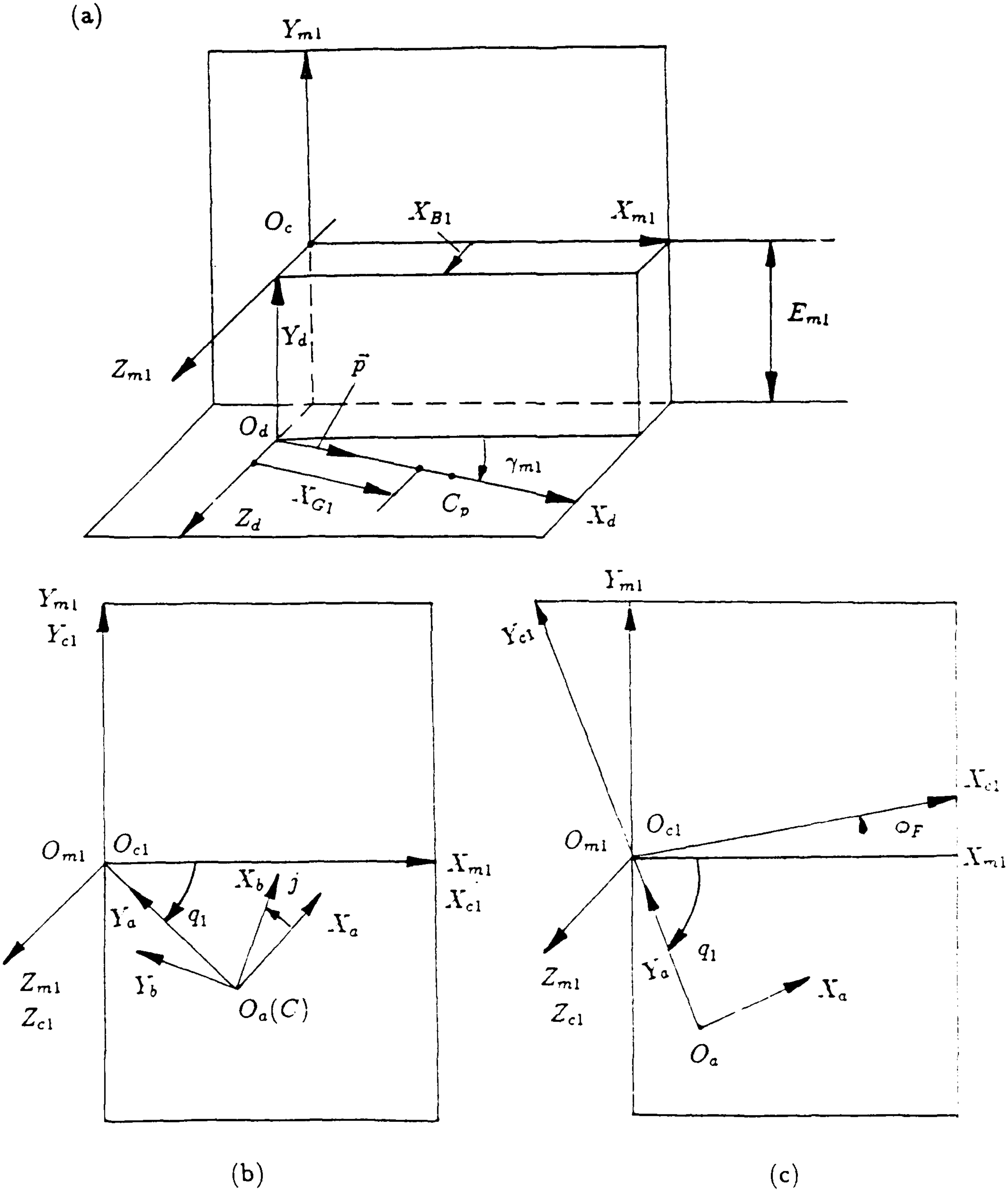


Fig. 2.1.1 Coordinate Systems and Pinion and Cradle Settings

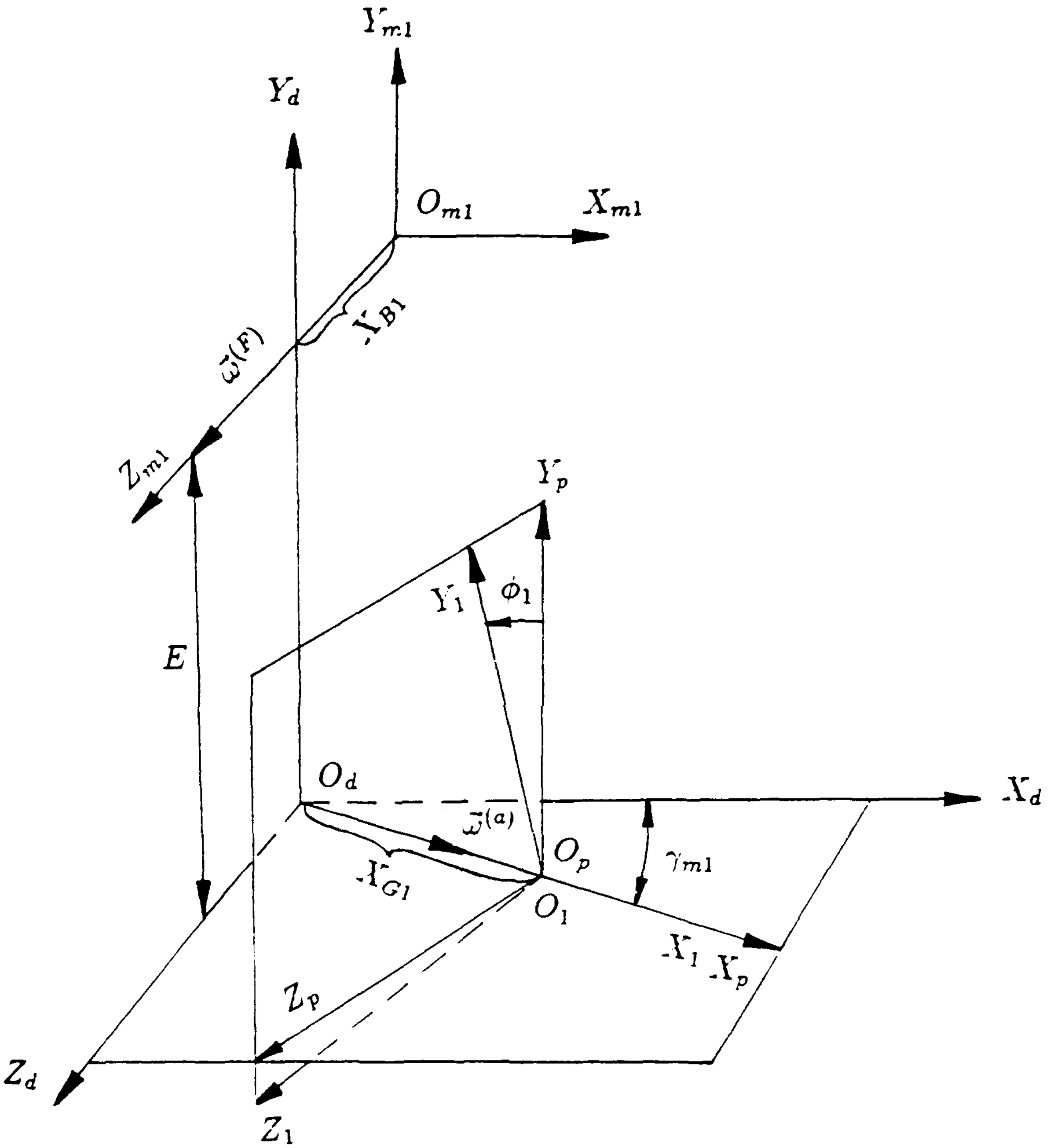


Fig. 2.1.2 Pinion Generation: Additional Coordinate Systems

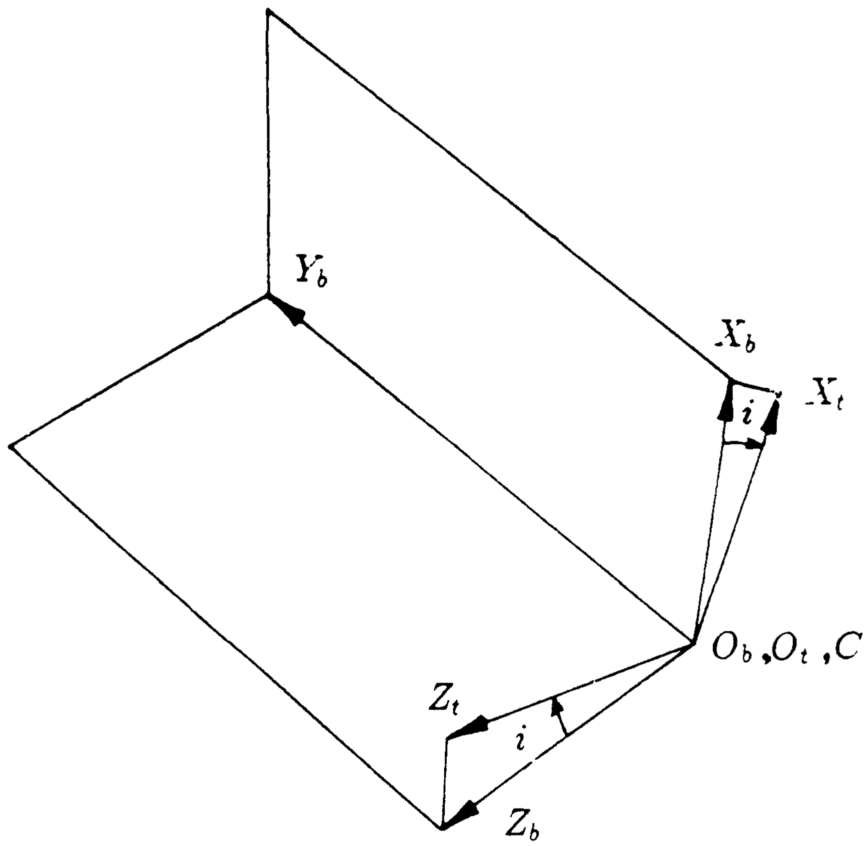
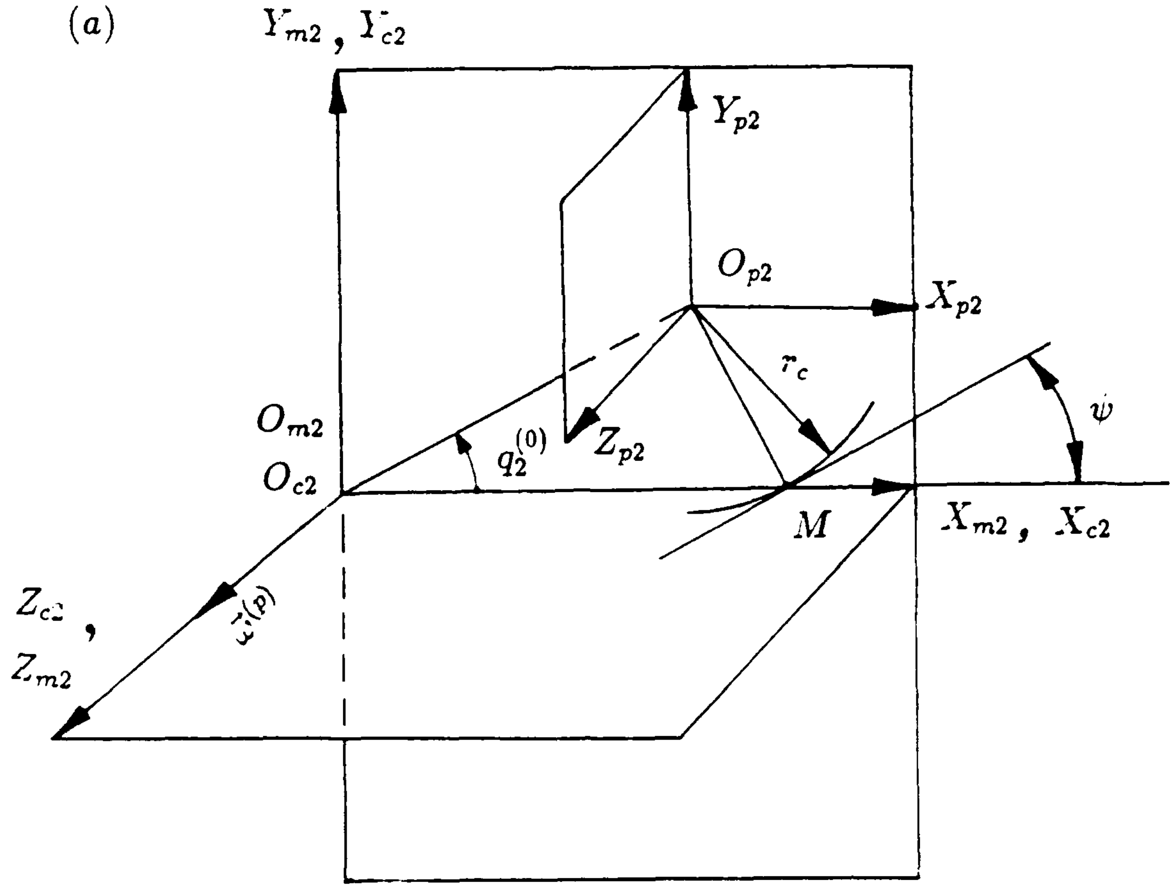


Fig. 2.1.3 Tilt of Pinion Head-Cutter



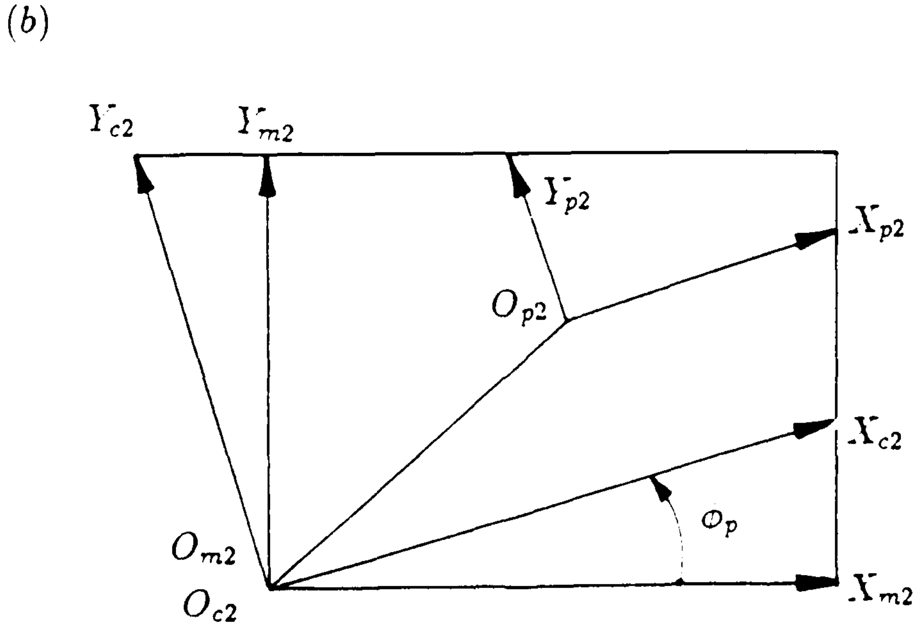


Fig. 2.2.1 Coordinate Systems and Gear and Cradle Settings

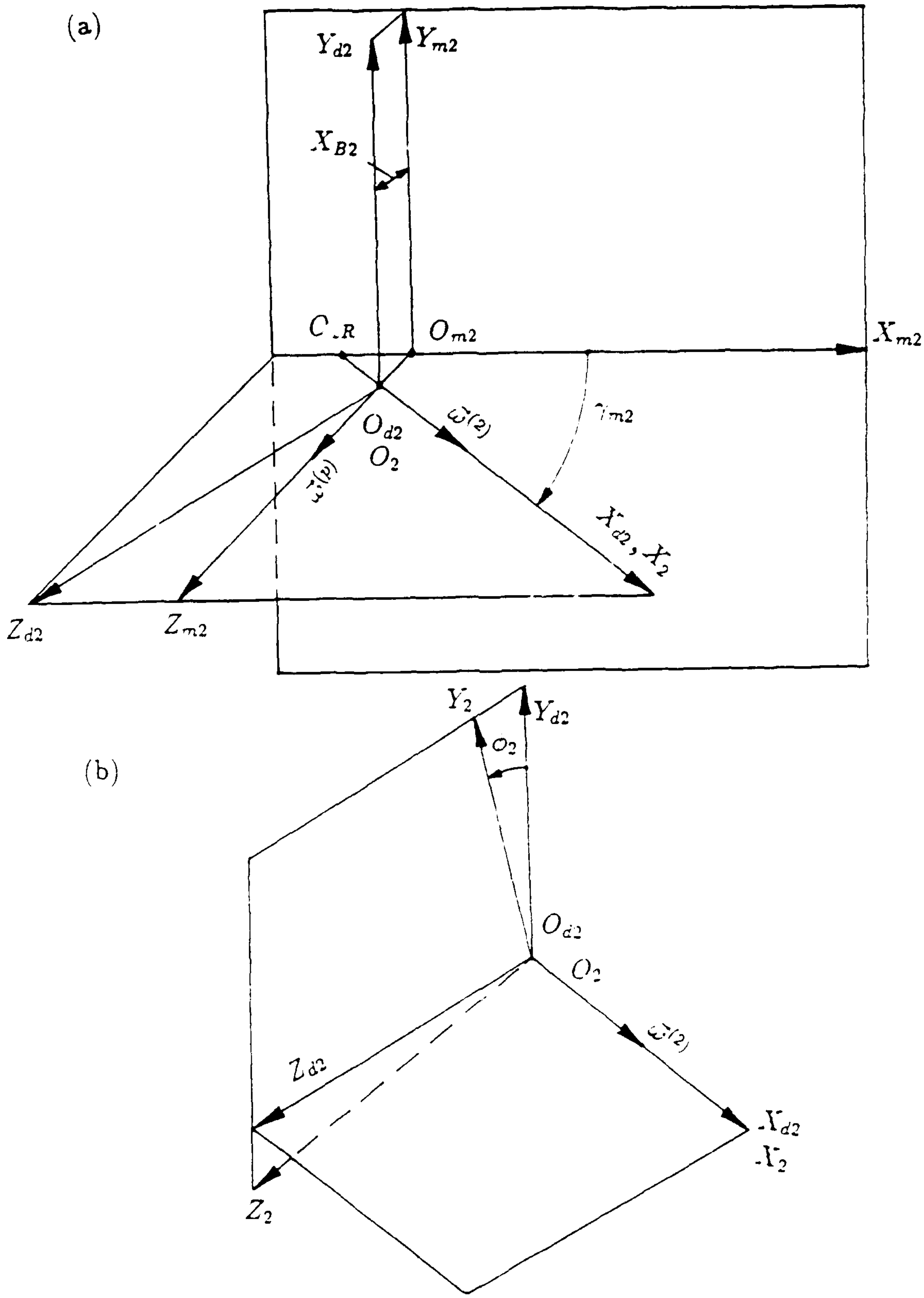


Fig. 2.2.2 Gear Generation: Additional Coordinate Systems

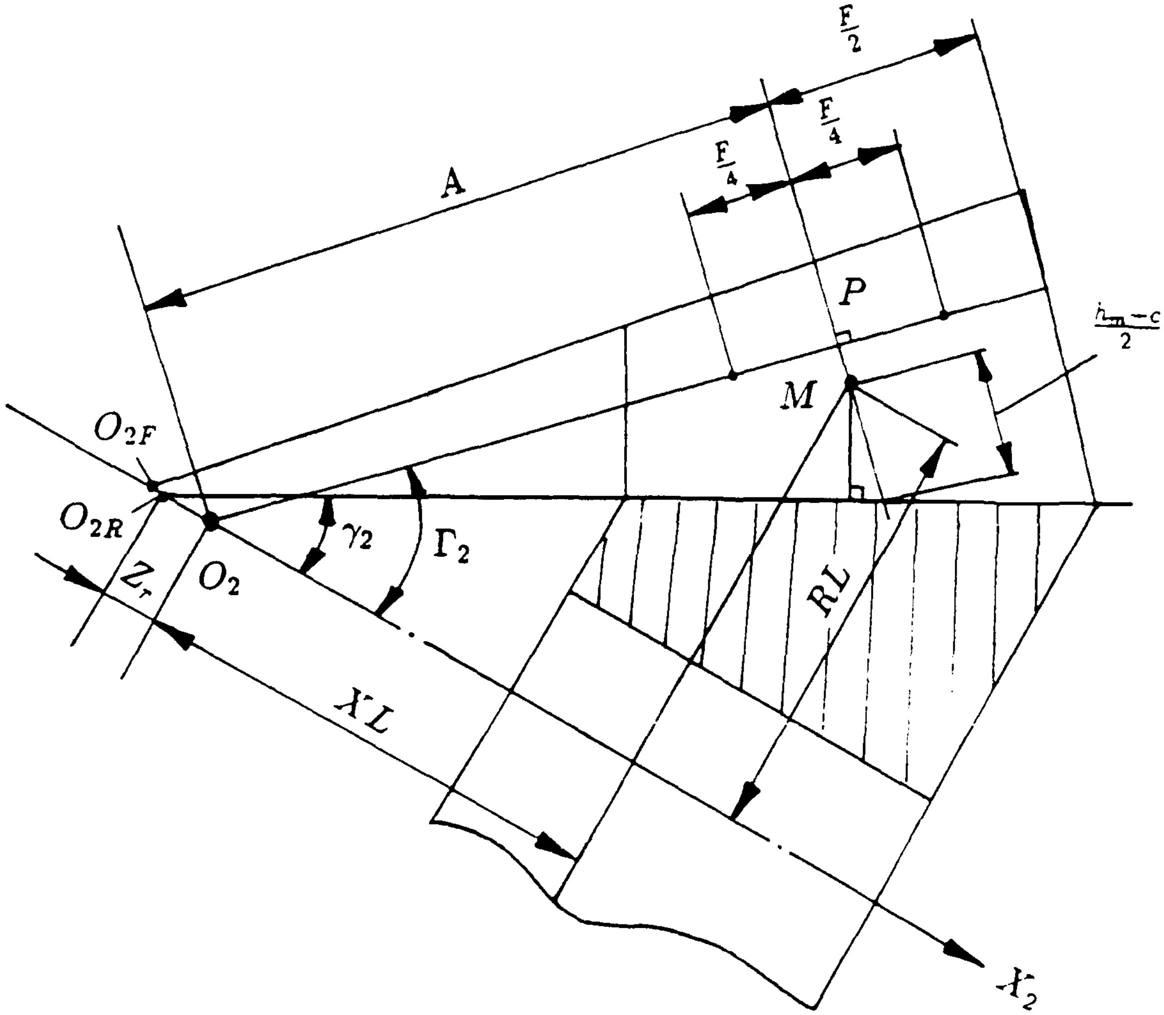


Fig. 2.3.1 Mean Contact Point

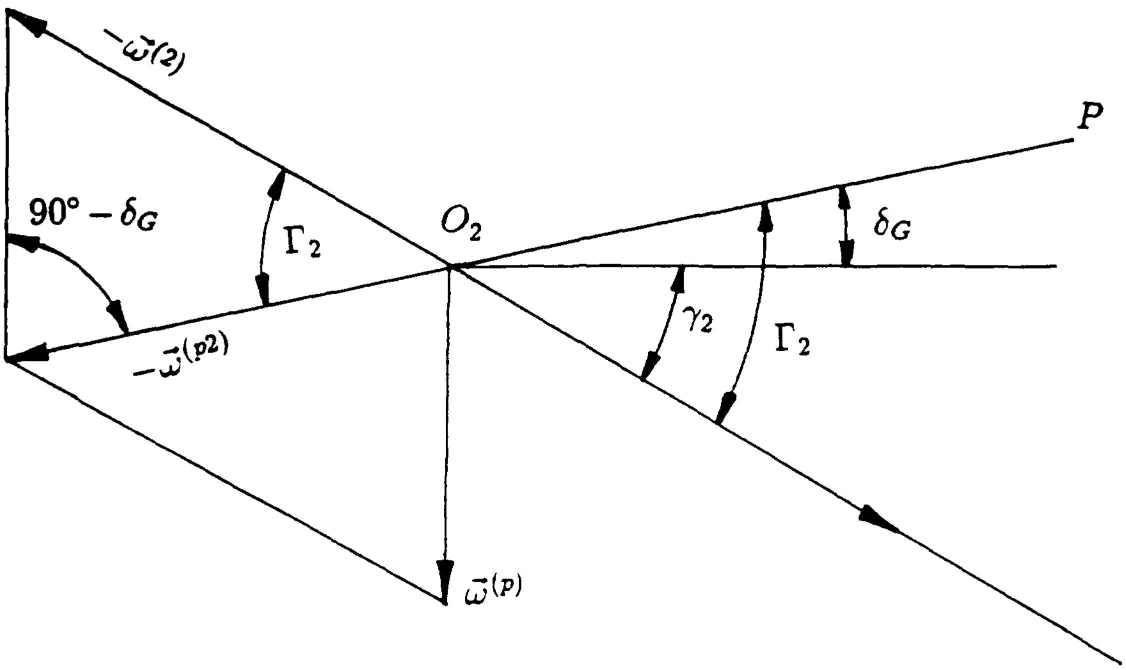


Fig. 2.3.2 Gear Generation: Instantaneous Axis of Rotation

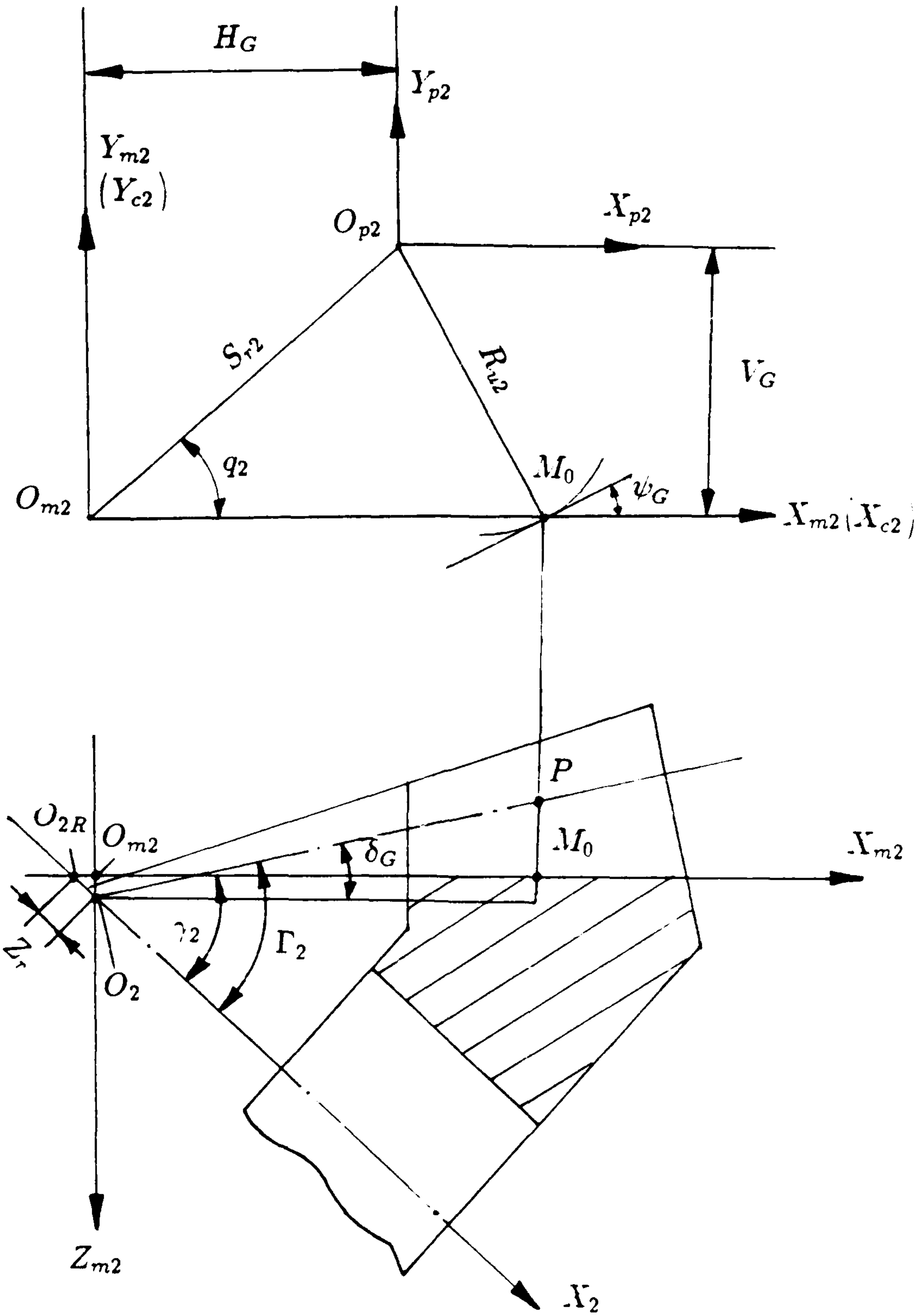


Fig. 2.3.3 Gear Head-Cutter Installments

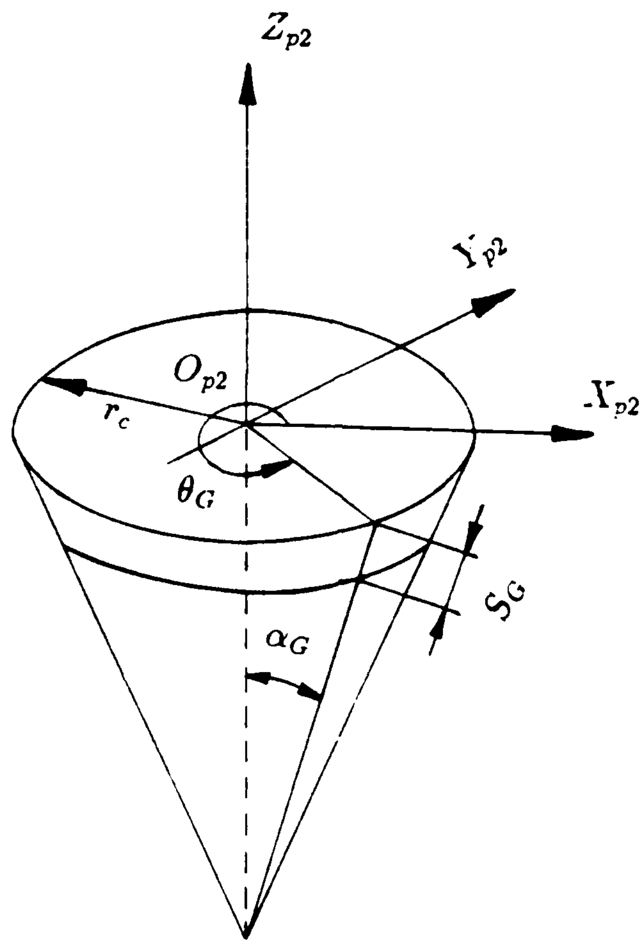


Fig. 3.1.1 Gear Head-Cutter

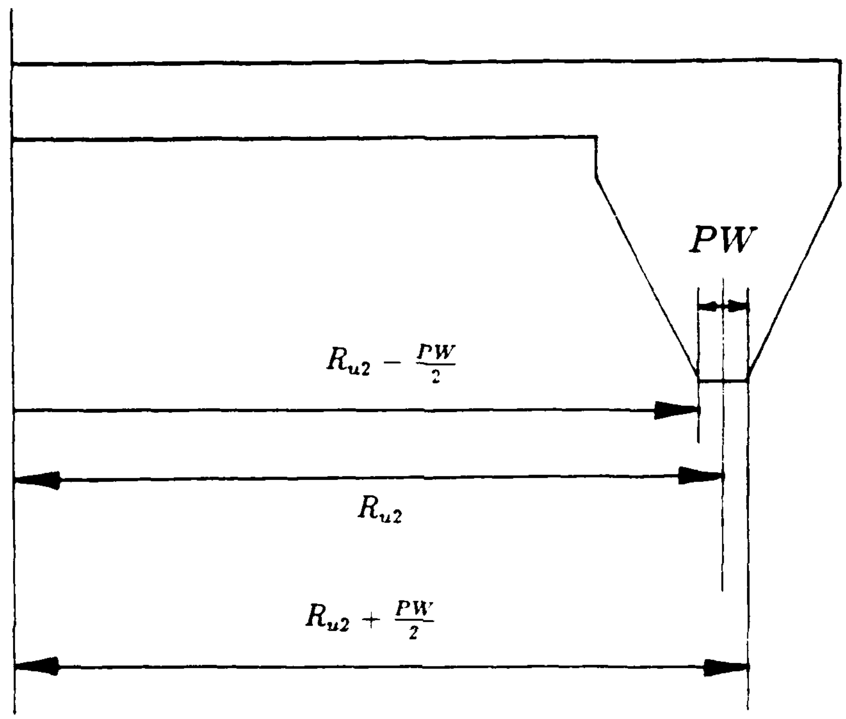


Fig. 3.1.2 Point Diameter

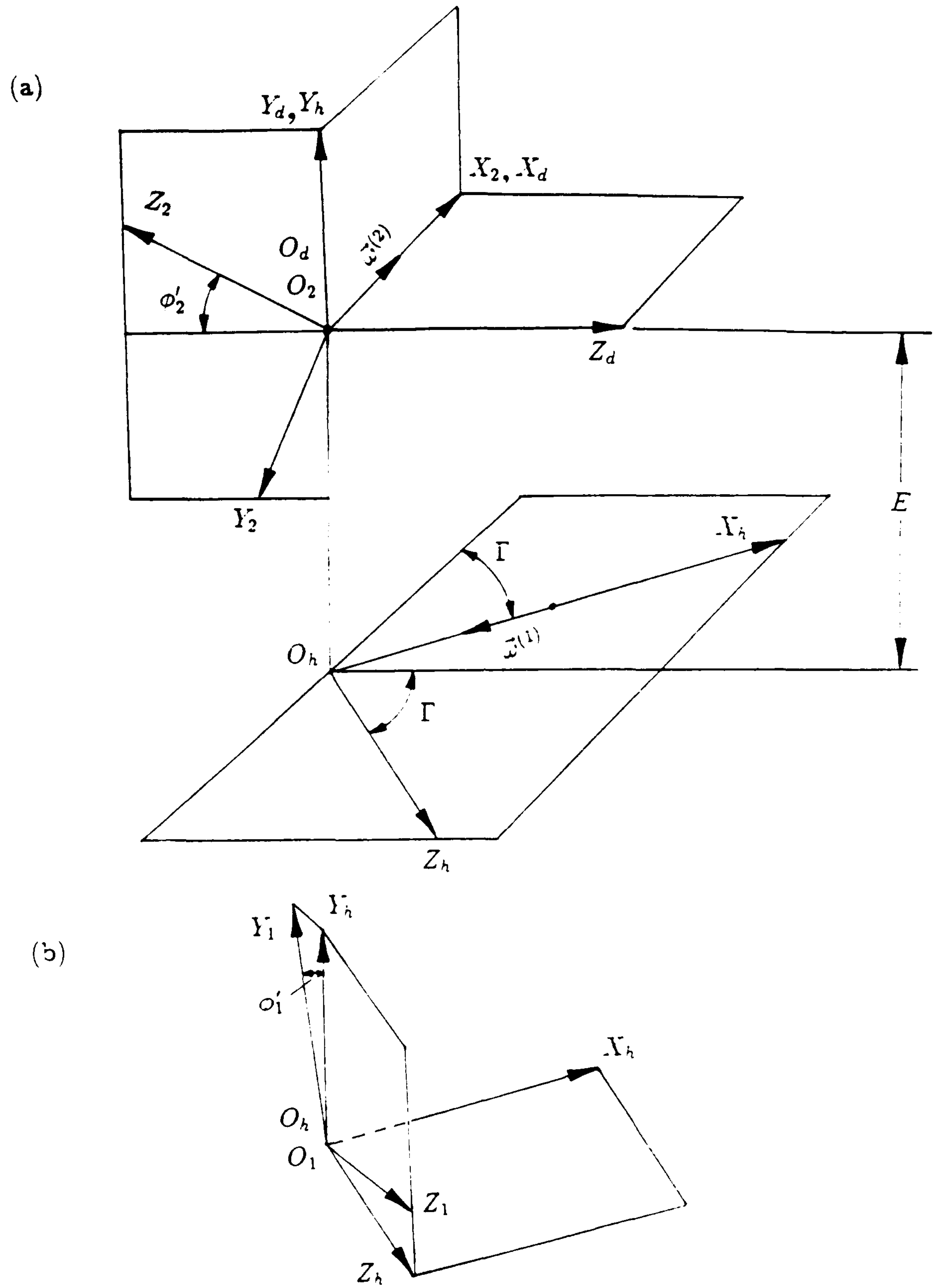


Fig. 4.2.1 Coordinate Systems for Simulation of Meshing

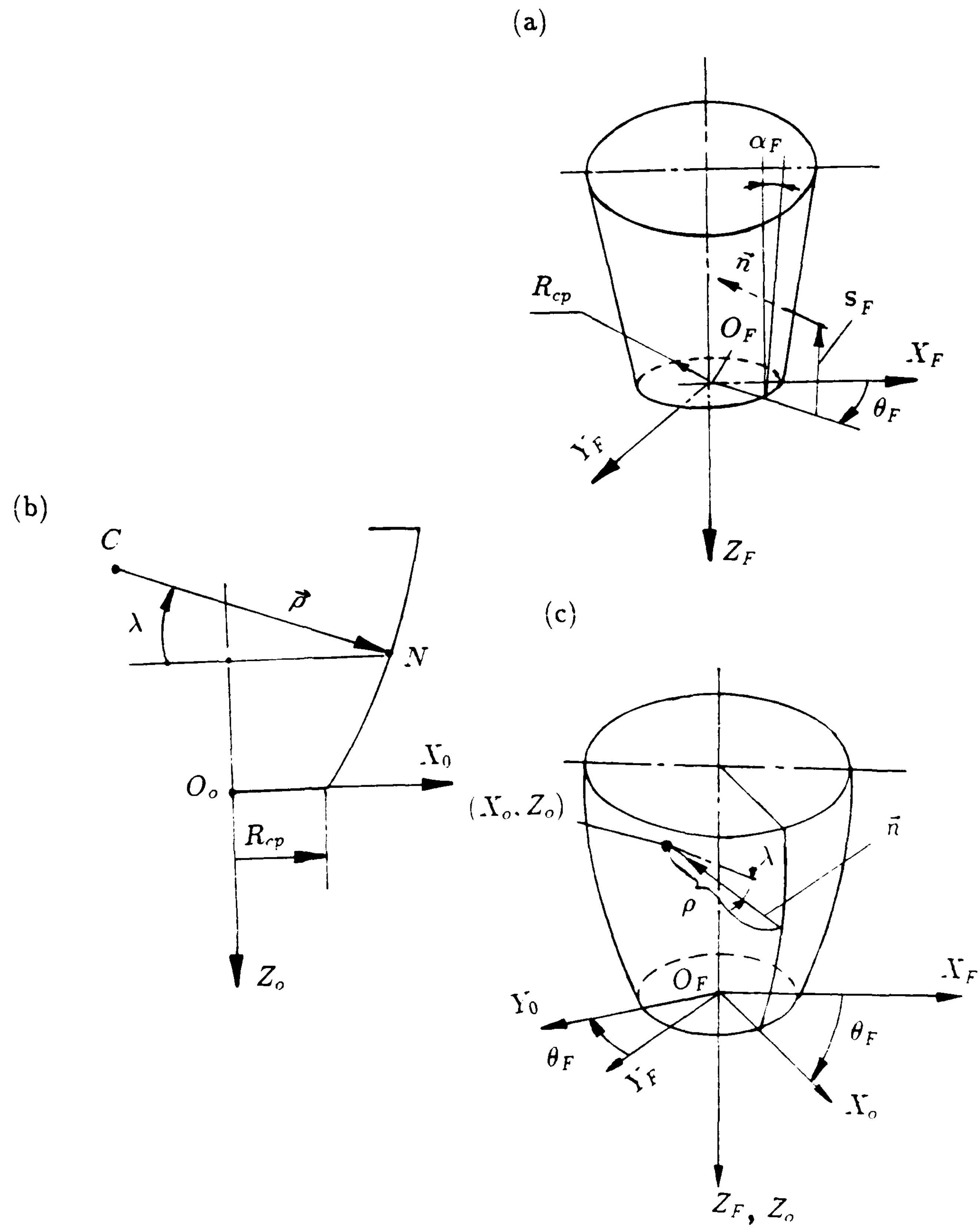


Fig. 5.2.1 Pinion Head-Cutters

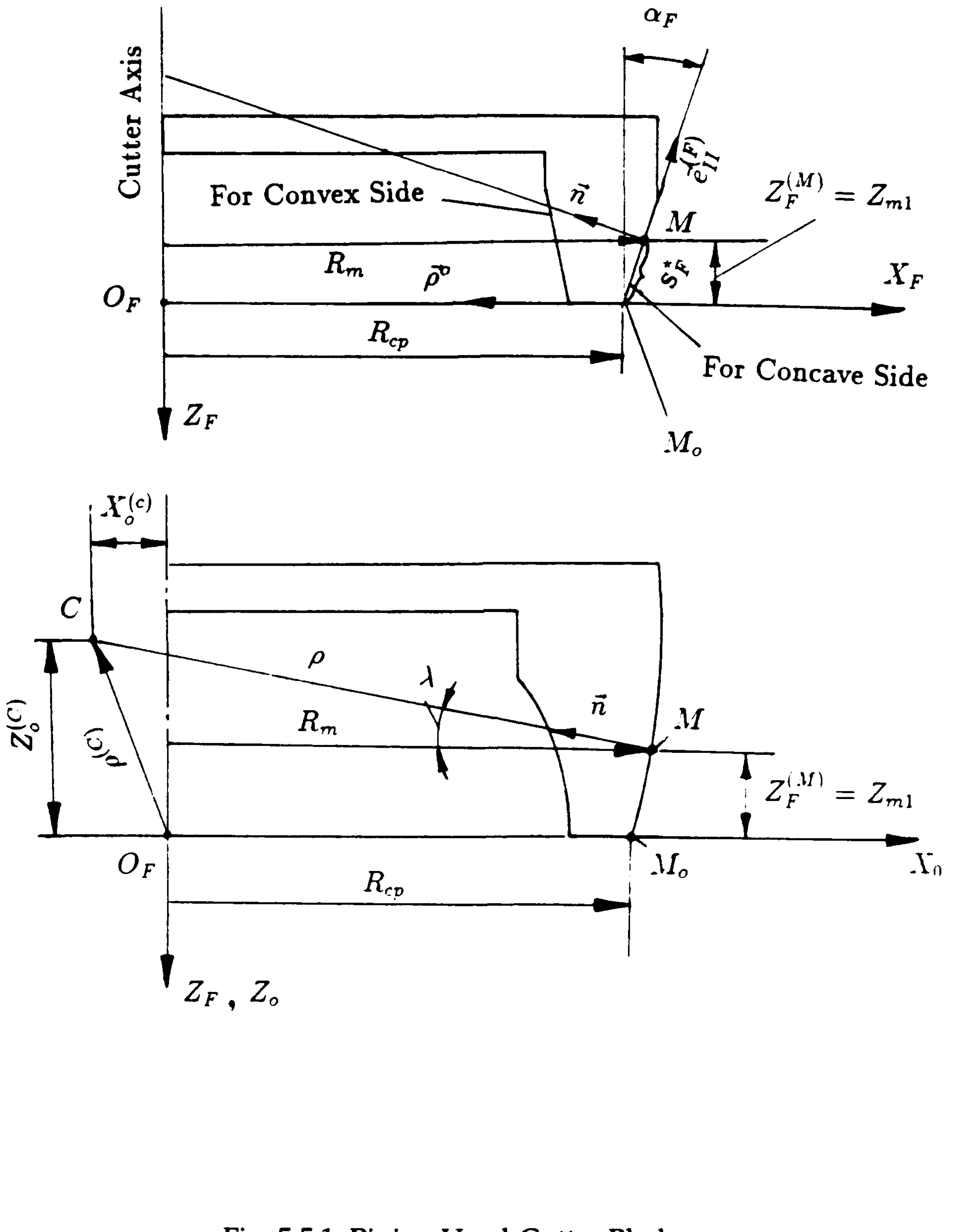


Fig. 5.5.1 Pinion Head-Cutter Blades

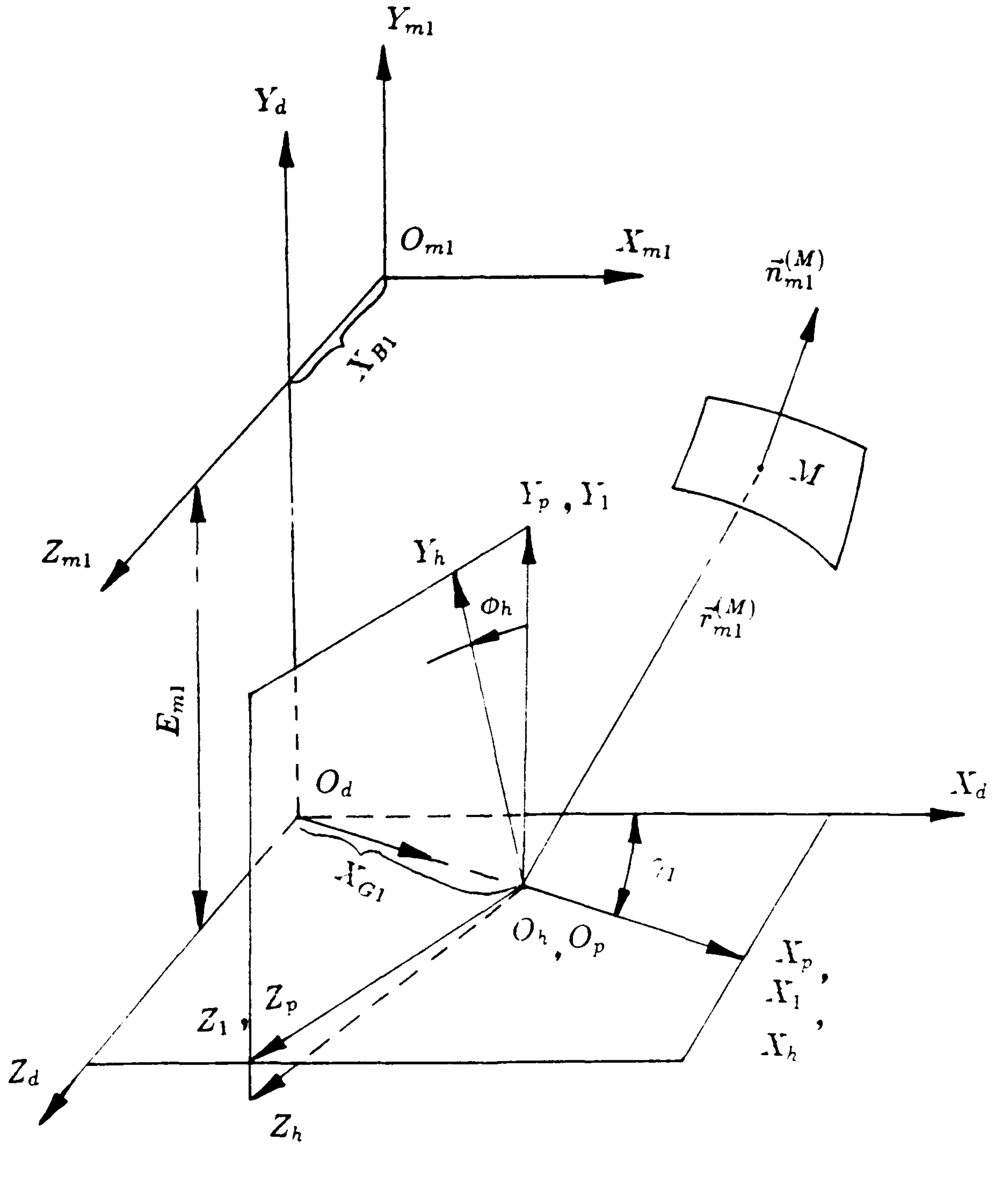


Fig. 5.2.2 Determination of O_h

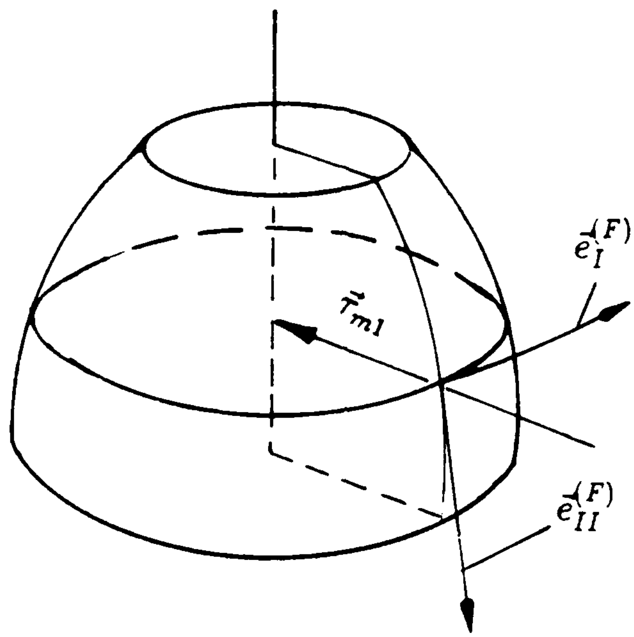


Fig. 6.3.1 Principal Directions of Pinion Head-Cutter Surface with Circular Arc Blades

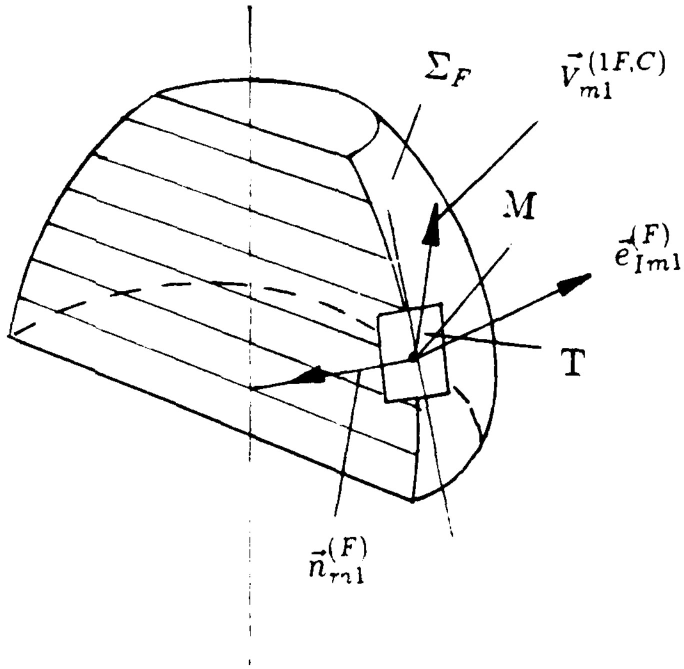


Fig.6.3.2 Visualization of Orientation of Vectors in Plane
Tangent to

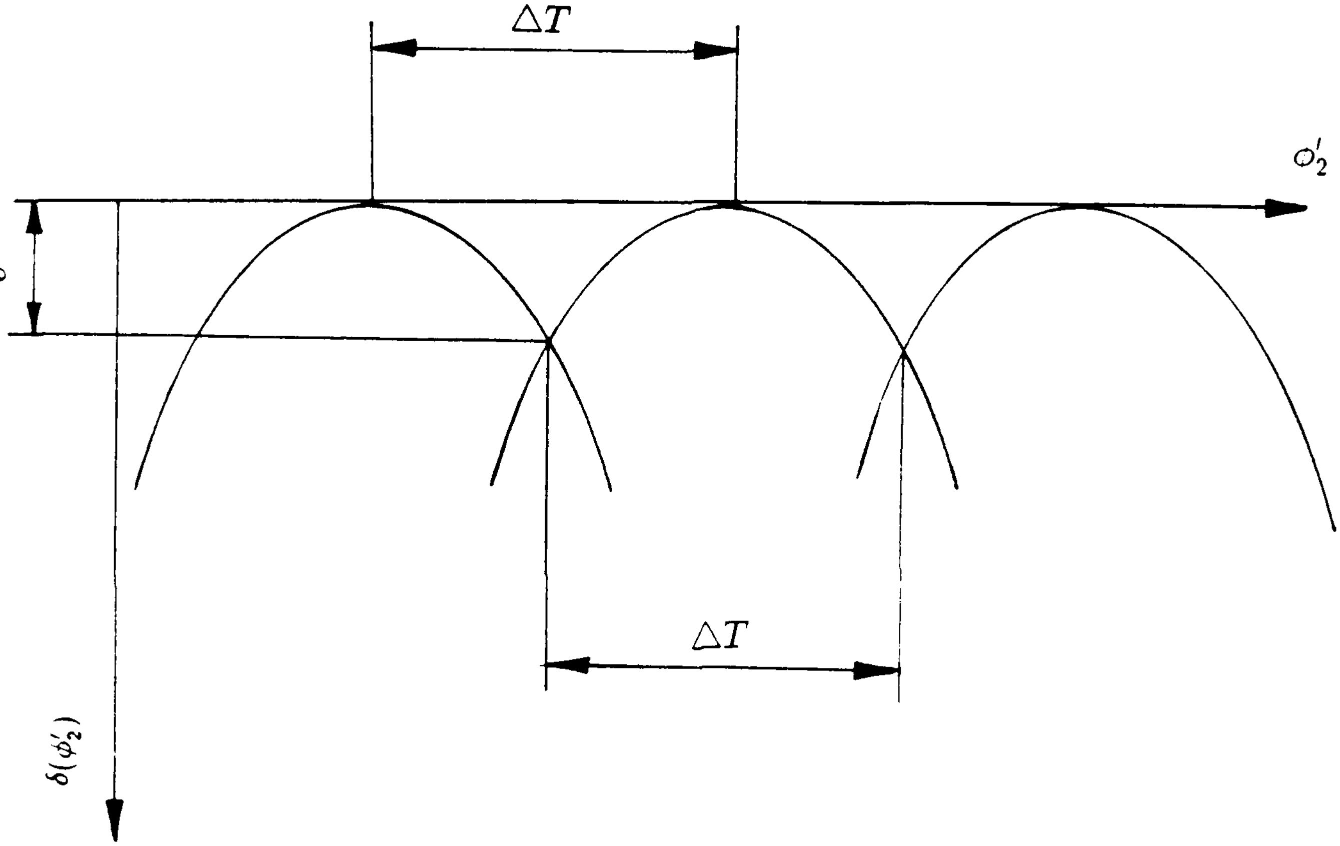


Fig. 6.4.1 Parabolic Function of Transmission Errors

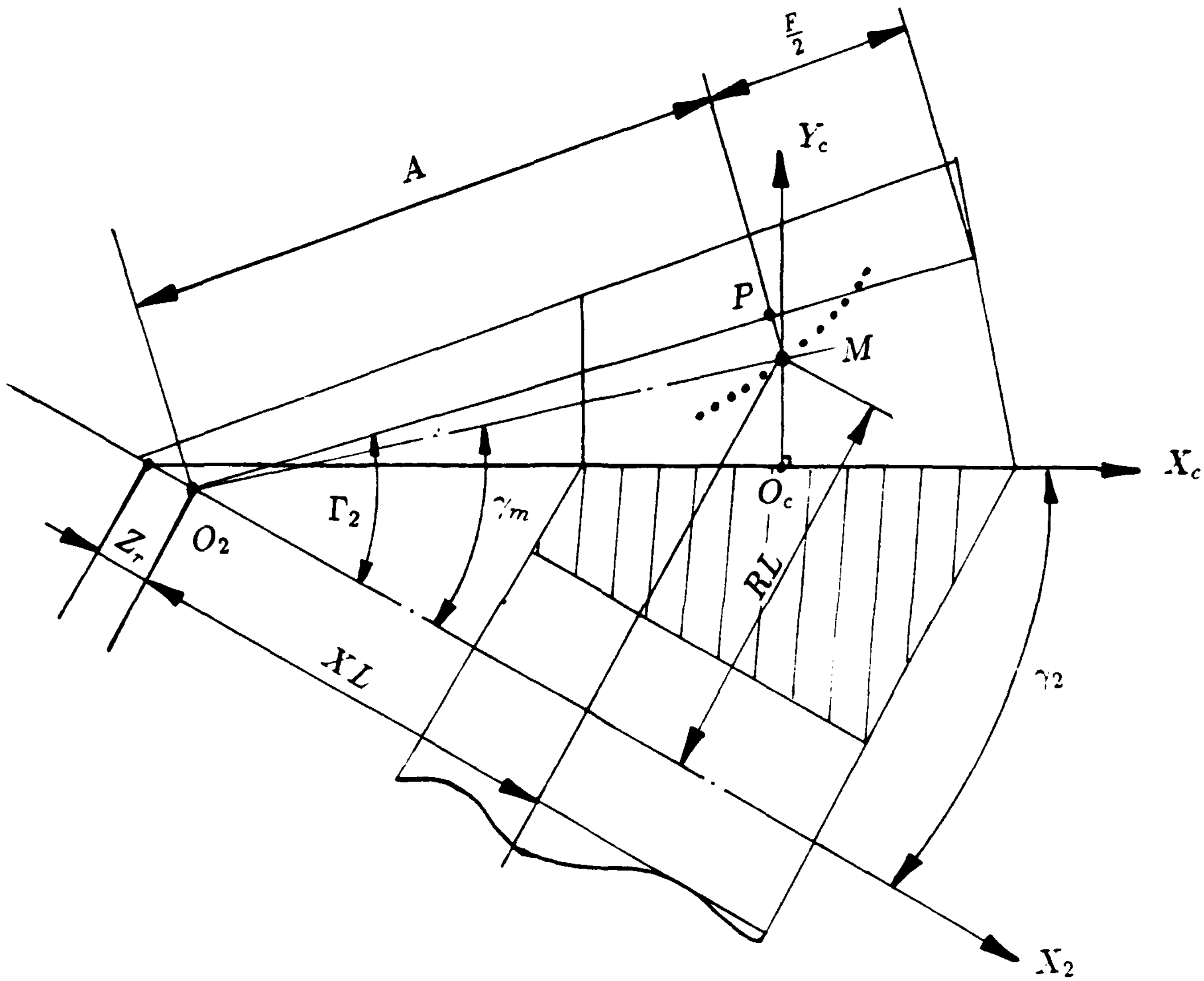


Fig. 6.5.1 Coordinate Systems Used for Visualization of Contact Path

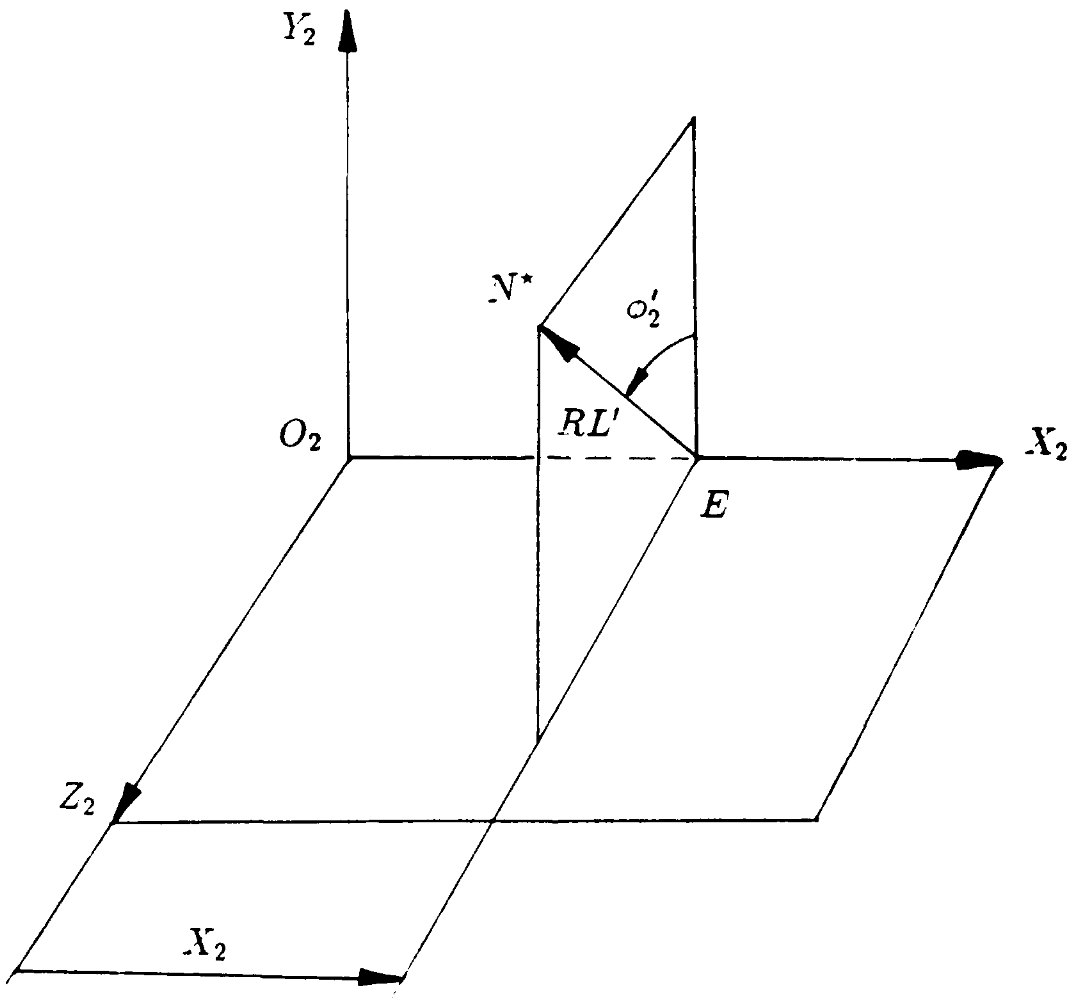


Fig. 6.5.2 Mapping of Contact Point N^*

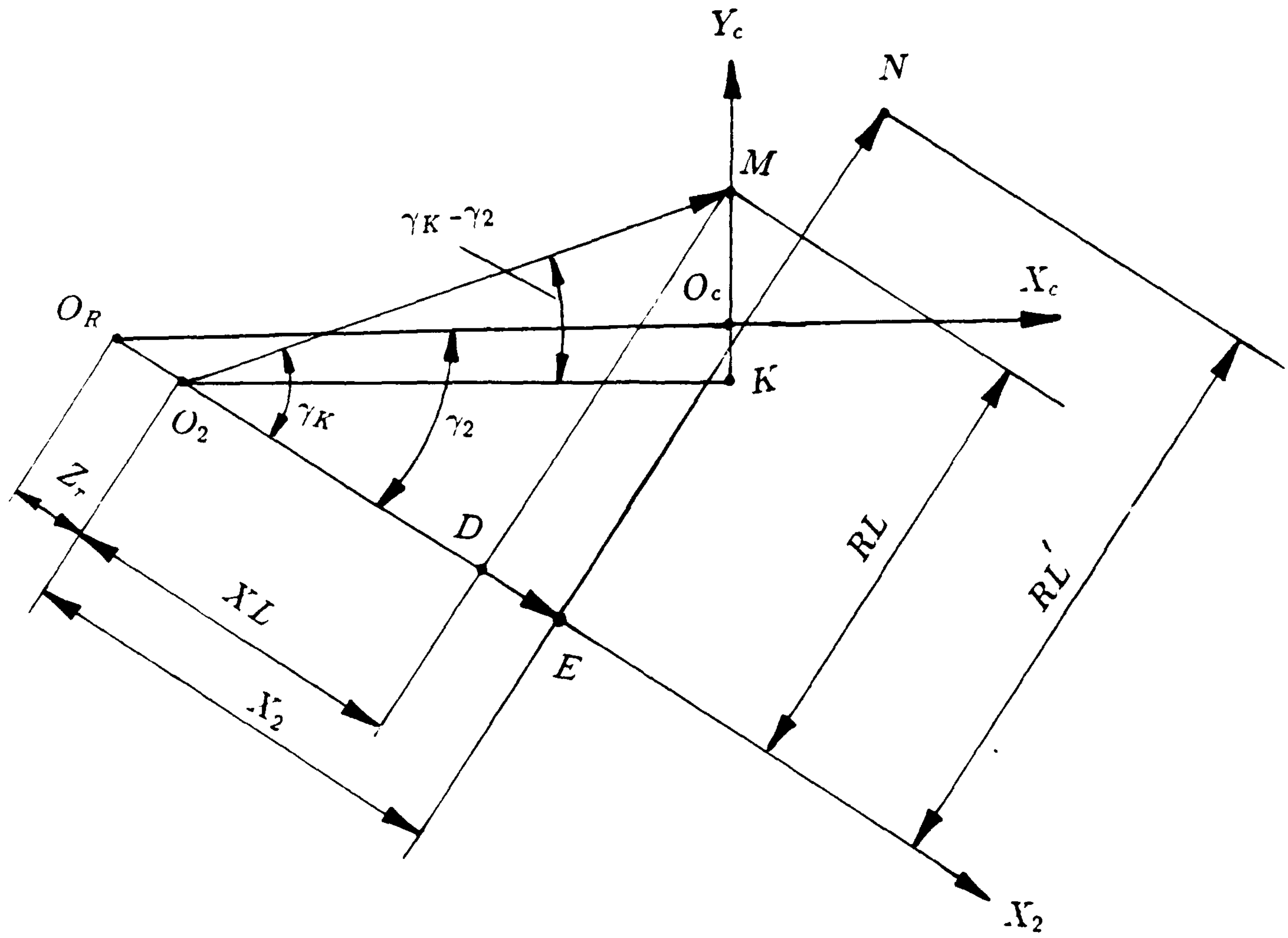


Fig. 6.5.3 Coordinates of Points of Contact M and N

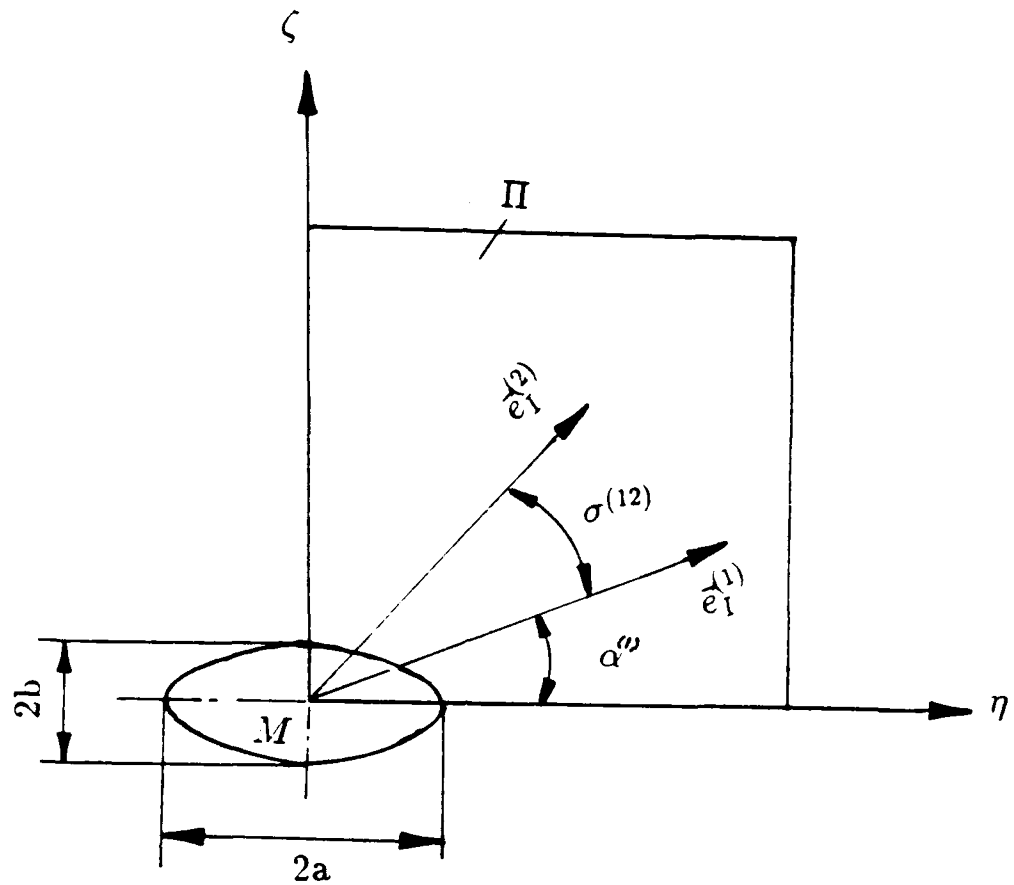


Fig. 6.5.4 Orientation of Contact Ellipse

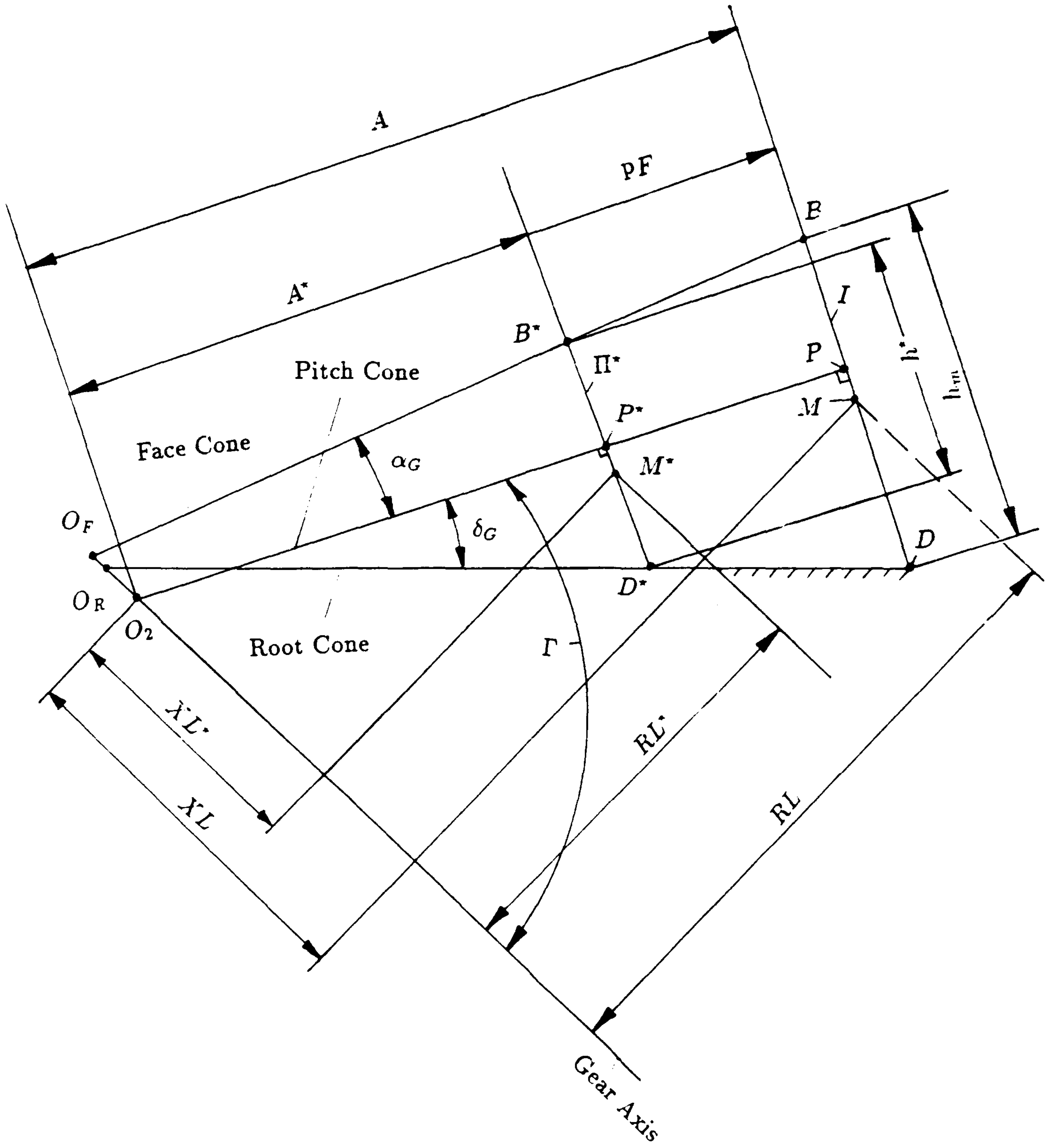


Fig. 7.1.1 Location of Mean Contact Point of Shifted Bearing Contact

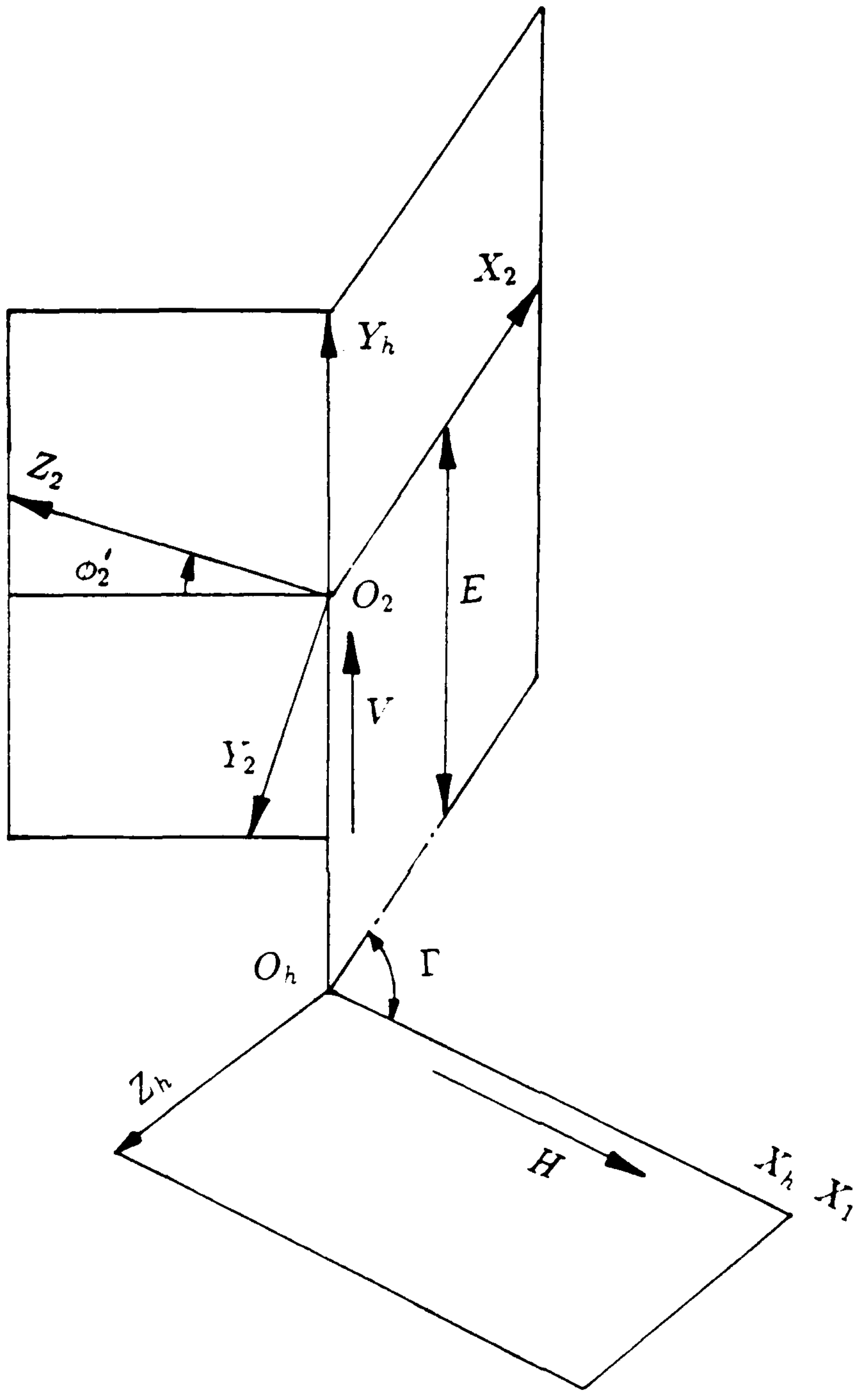


Fig. 7.1.2 Gear-Pinion Misalignment

CAM PROFILE AND ITS MOTION

b = 5.5, E =15, r_u = 3.6230

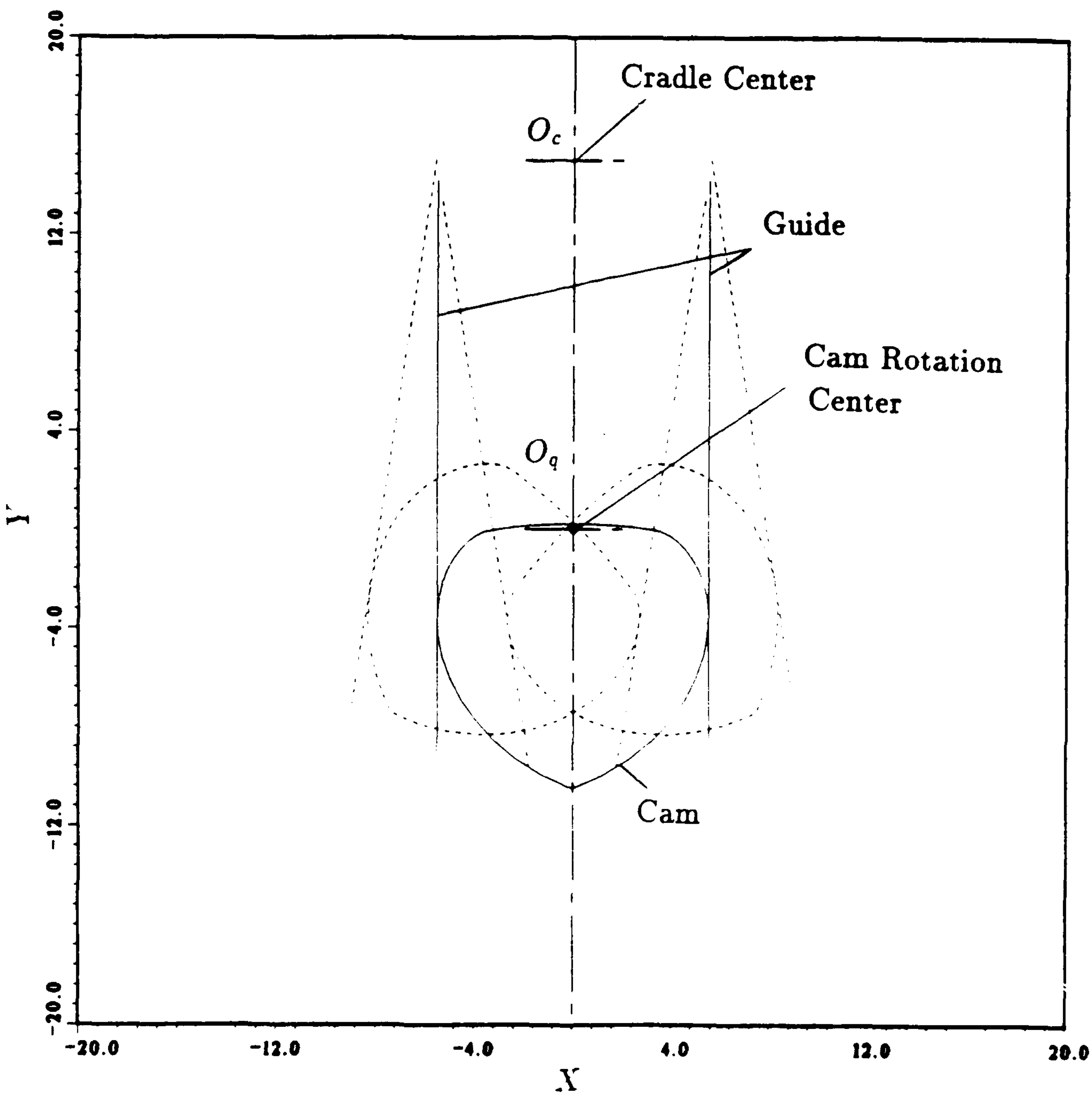


Fig. A.3.1 Cam Profiles and Guides

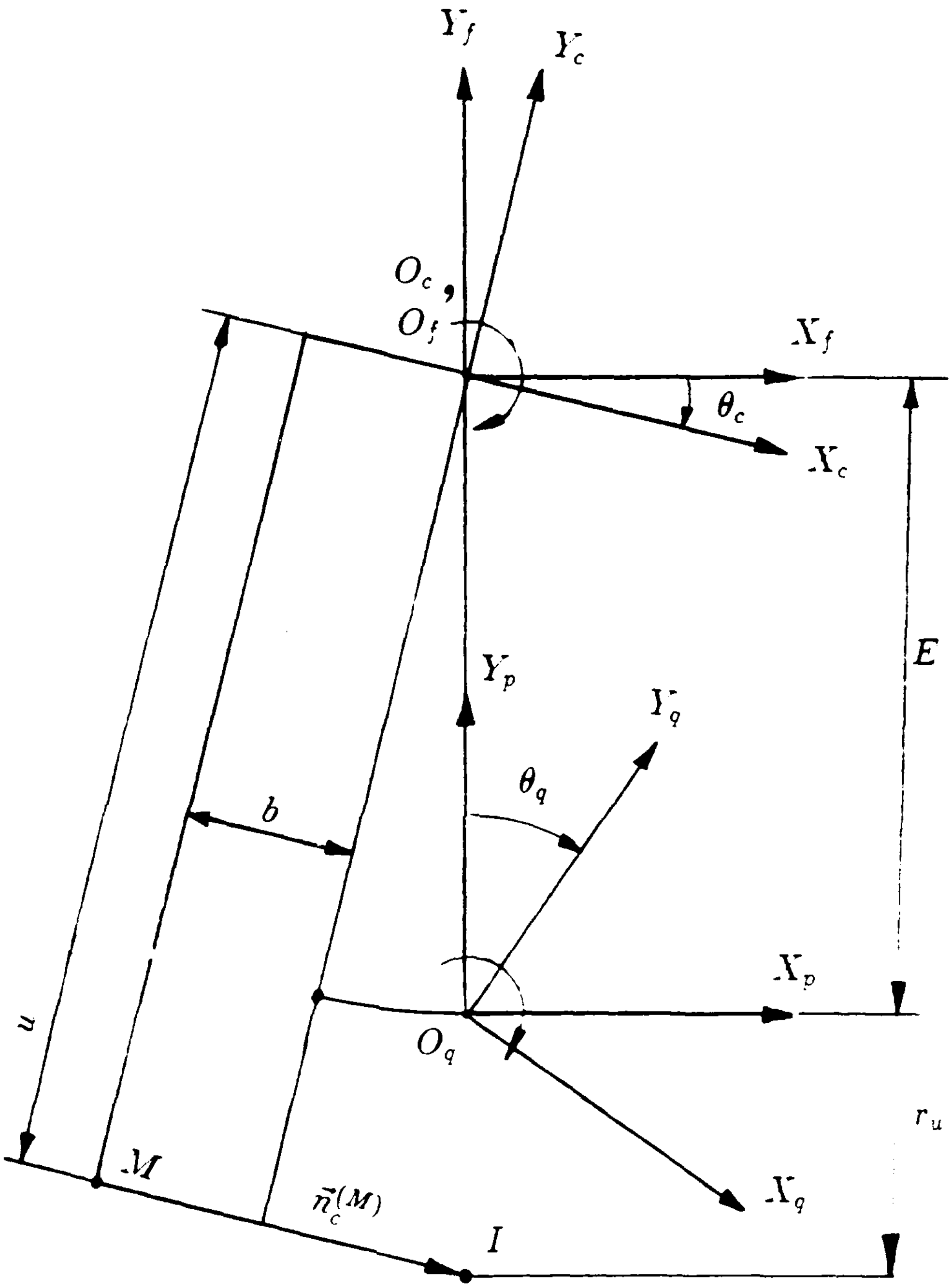


Fig. A.3.2 Schematic of Motions of Cam and Guides without Modified Settings

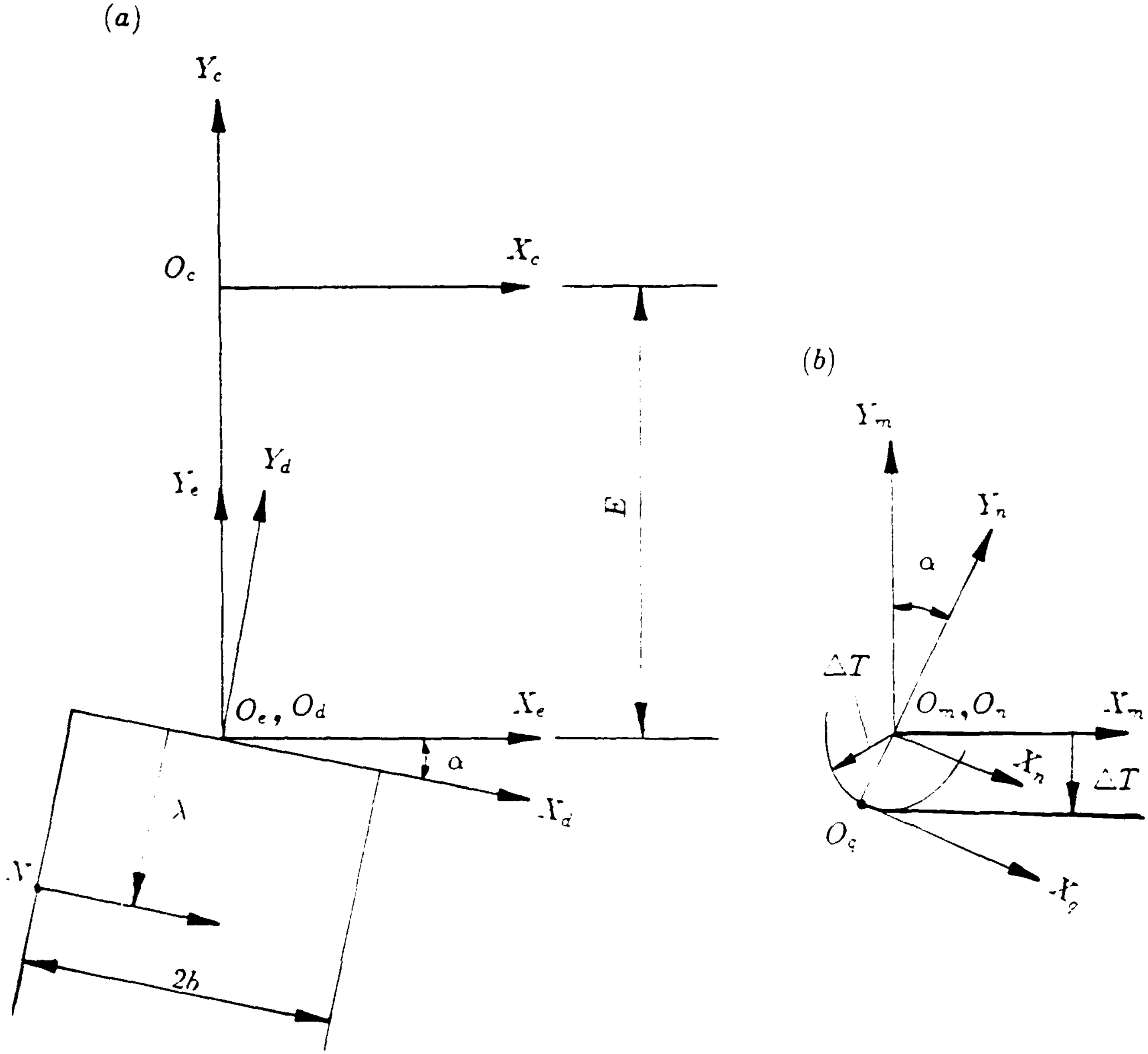


Fig. A.4.1 Coordinate Systems for Cam Mechanism with Modified Settings

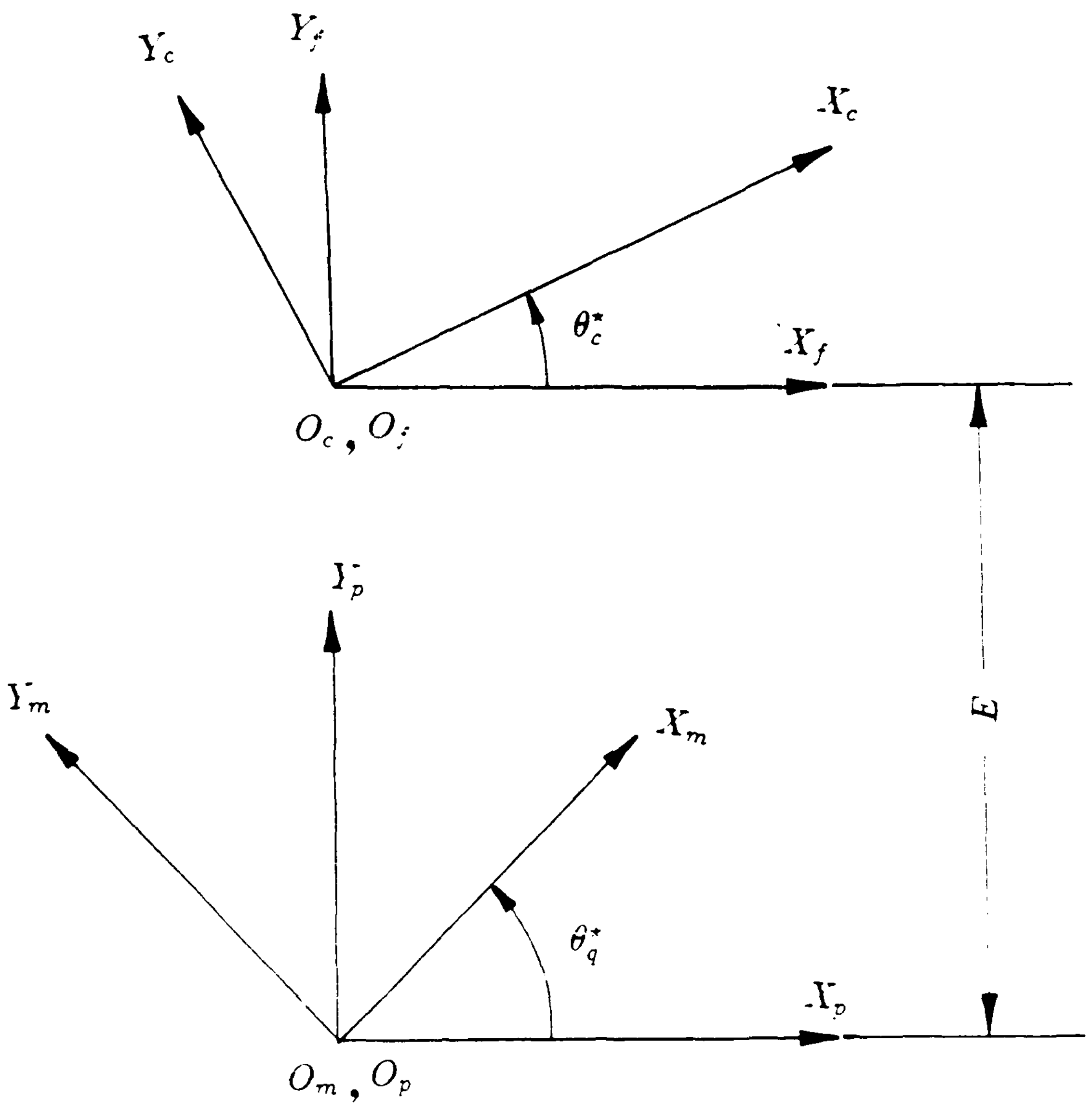
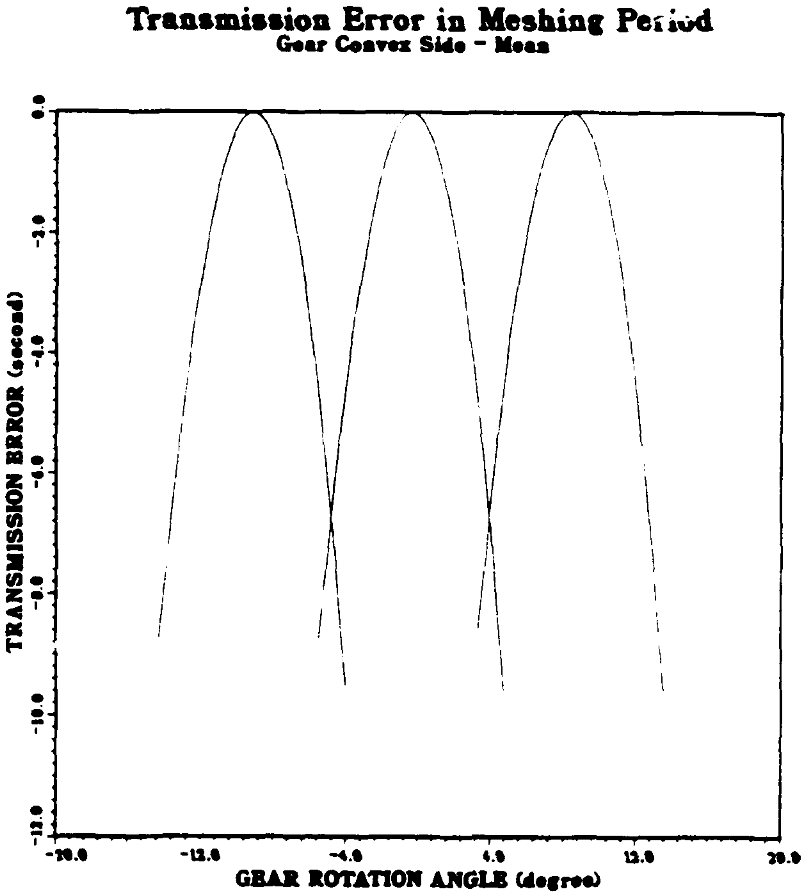


Fig. A.4.2 Transmission of Cam Rotation into Cradle Rotation by Modified Settings



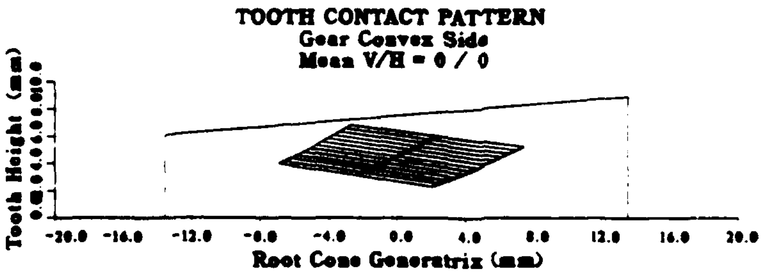
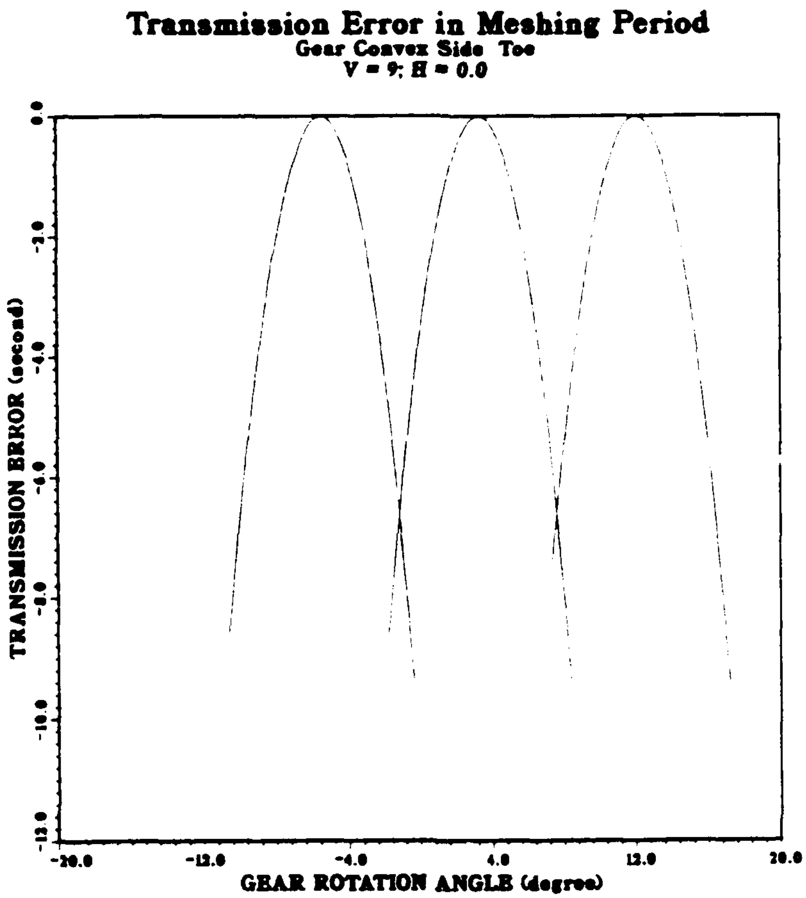


Fig. B.1 Contact Pattern and Transmission Errors at Gear Mean Position of Convex Side



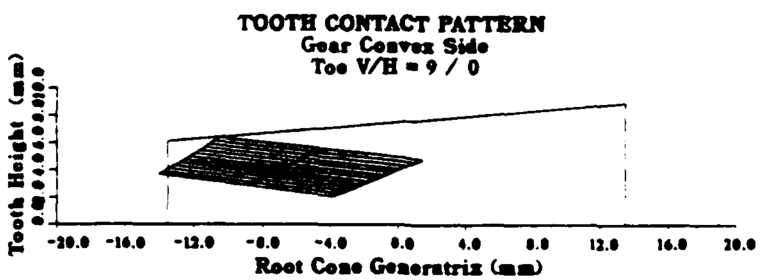
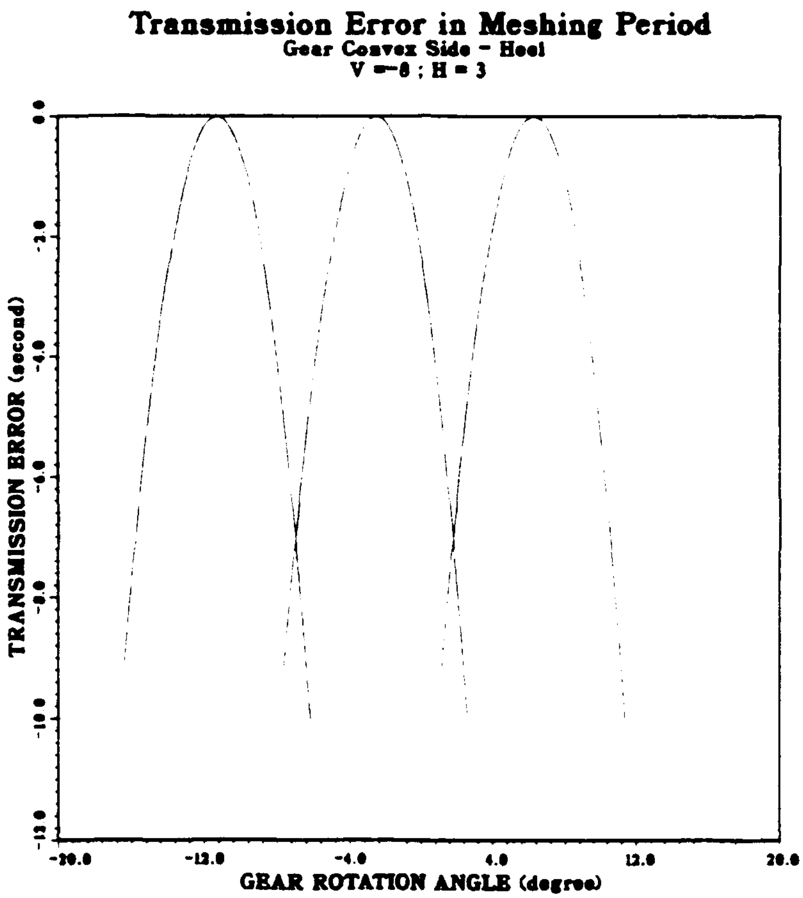


Fig. B.2 Contact Pattern and Transmission Errors at Gear Toe Position of Convex Side



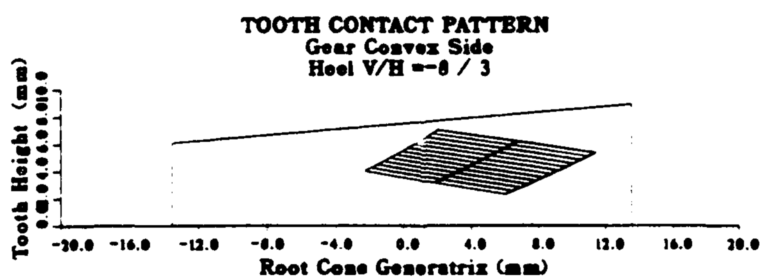
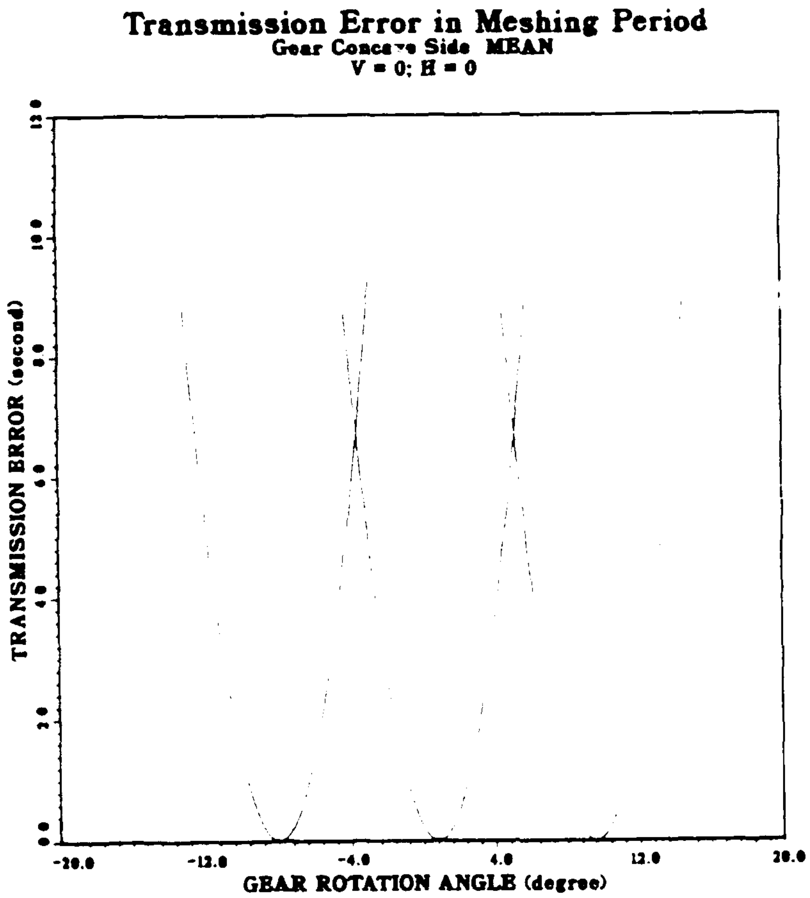


Fig. B.3 Contact Pattern and Transmission Errors at Gear Heel Position of Convex Side



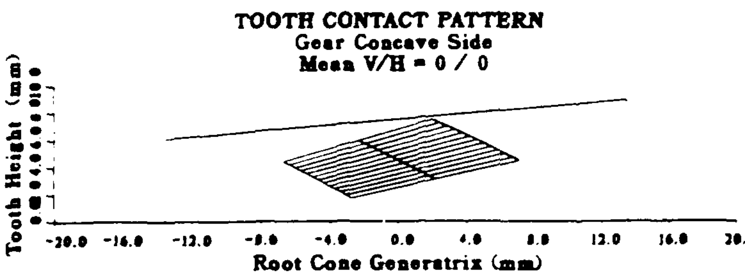
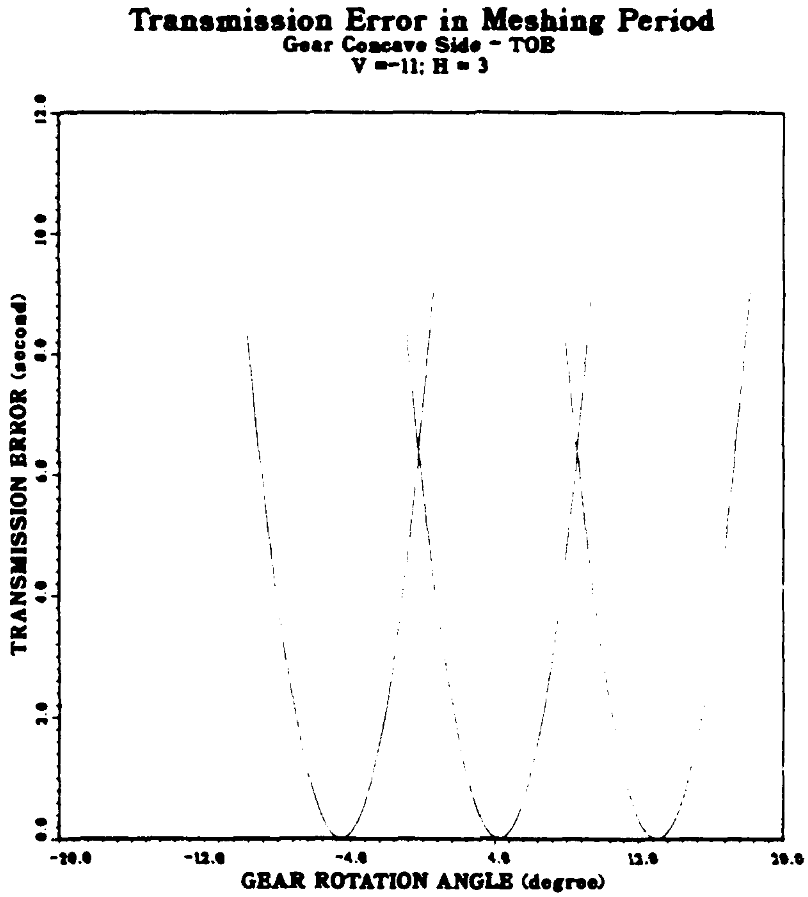


Fig. B.4 Contact Pattern and Transmission Errors at Gear Mean Position of Concave Side



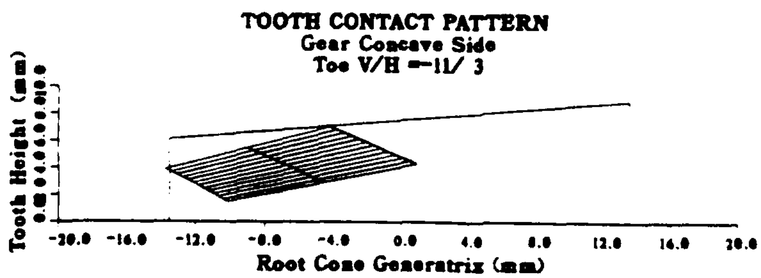
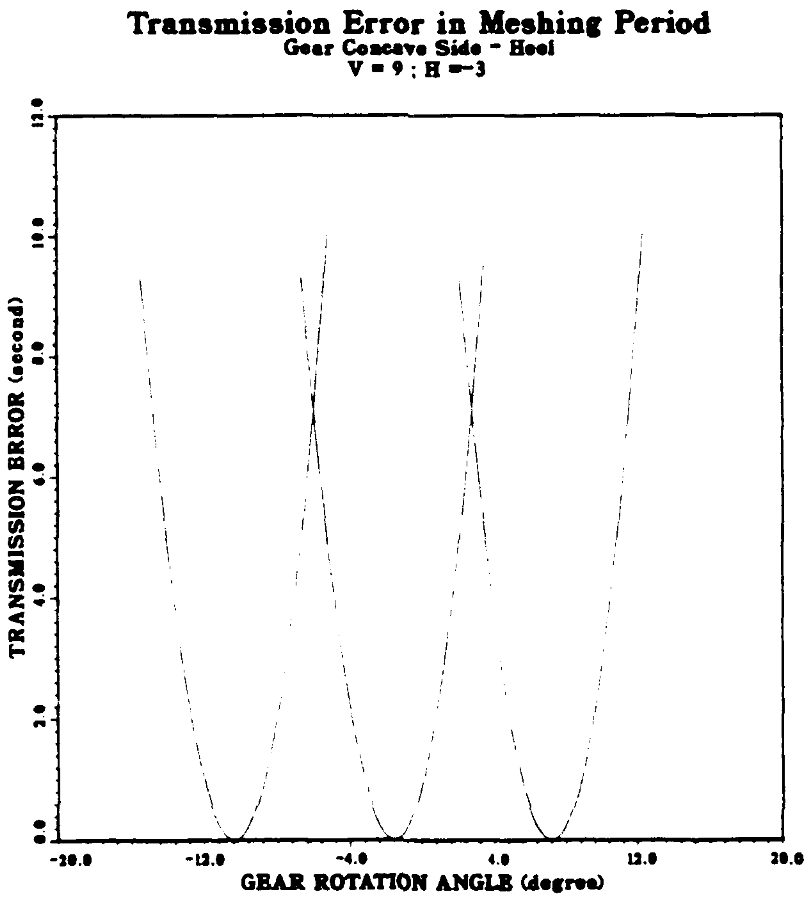


Fig. B.5 Contact Pattern and Transmission Errors at Gear Toe Position of Concave Side



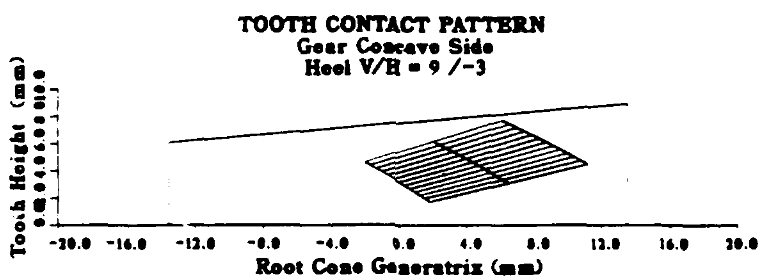


Fig. B.6 Contact Pattern and Transmission Errors at Gear Heel Position of Concave Side

```
C...
C....     THIS PROGRAM IS TO DERIVE THE MACHINE TOOL SETTINGS
C....       FOR PINION GENERATION & TEST THE RESULTS
C...
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 KS,KQ,K1I,K1II,K2I,K2II,KFI,KFII,KD,KF,KH,mcd
      REAL*8 M11,M12,M13,L11,L12,L13,L14,M21,M22,M23,L21,L22,L23,L24,
     &N11,N12,N21,N22
      real*8 xi(5),x(5),f(5)
      EXTERNAL FCN1,FCN2,FCN,FCNM,FCNR,FCNMR
      DIMENSION CH(3),P(3),E1EF(3),ESN(3),EQN(3),W1VT2(3),WV12(3),
     $W2VT1(3),EFIH(3),EFIIH(3),RH(3),GNH(3),E2IH(3),E2IIH(3),PI2P(20),
     &E1IH(3),E1IIH(3),EFI(3),EFII(3),E1I(3),E1II(3),GN(3),EFE1(3),
     &ERR(20),xcp(20),ycp(20),AX1(20),AX2(20),ANG1(20),ANG2(20)
      COMMON/A1/CNST,TN1,TN2,C,FW,GAMMA,x1,r1,mcd
      COMMON/A2/B1,RGMA1,FGMA1,PGMA1,D1R,D1F,ADD1,DED1
      COMMON/A3/B2,RGMA2,FGMA2,PGMA2,D2R,D2F,ADD2,DED2,WD,CC,D2P
      COMMON/A4/SR2,Q2,RC2,PW2,XB2,XG2,EM2,GaMA2,CR2,ALP2,PHI2,PHI2P
      COMMON/A5/SG,XM,YM,ZM,XNM,YNM,ZNM,X2M,Y2M,Z2M,XN2M,YN2M,ZN2M,
     &XNH2,YNH2,ZNH2,XH2,YH2,ZH2
      COMMON/A6/ES(3),EQ(3),CN(3),W1(3),W2(3),W12(3),VT1(3),VT2(3),
     $V12(3),KS,KQ,KF,KH,EF(3),EH(3),SIGSF,PI21
      COMMON/A7/SR1,Q1,Rcf,PW1,XB1,XG1,EM1,GaMA1,CR1,ALP1,PHI1,PHI1P
      COMMON/A8/Sf,XM1,YM1,ZM1,XNM1,YNM1,ZNM1,X1M,Y1M,Z1M,
     &XN1M,YN1M,ZN1M,XNH1,YNH1,ZNH1,XH1,YH1,ZH1
      COMMON/A9/PHI2P0,OX,OZ,XO,ZO,RHO,ALP,V,H,CR1T,PCR1T
      COMMON/A10/K1I,K1II,K2I,K2II,DEL,E1IH,E1IIH,E2IH,E2IIH,GNH,
     &A2P,B2P,TAU1R,TAU2R,A2L,B2L
      COMMON/A11/RAM,PSI1,C2,D6,E24,F120,CX6,DX24,EX120,RU1,DELT,RUP,
     $RA1,CPF,DPF,EPF,FPF
      CNST=DARCOS(-1.0D00)/180.0D00
C
C...
C...  INPUT THE CONTROL CODES
C...
C
C     IF V AND H CHECK IS NOT DESIRED, SET JCN = 1
C     DO NOT SET JCN TO BE 3
C
      JCL=2
C
C...  FOR RIGHT HAND GEAR JCH=1, FOR LEFT HAND GEAR JCH =2
C
      JCH=1
C
C...  FOR STRAIGHT BLADE JCC=1, FOR CURVED BLADE JCC=2
C
      JCC=1
C
C     TL1 AND TL2 ARE NUMBER OF EXTRA POINT ON CONTACT PATH
C      WHICH SHOULD NOT BE LARGER THAN 2
C
      TL1=1.0
      TL2=1.0
C...
C...  INPUT BLANK DATA OF GEAR AND PINION
C...
      TN1=11.0
      TN2=41.0
      C=0.0
```

```
      FW=27.03
      GAMMA=90.0*CNST
      MCD=76.56
      RGMA1=13.3333*CNST
      B1=35.0*CNST
      B2=35.0*CNST
      RGMA2=70.6500*CNST
      FGMA2=76.6667*CNST
      PGMA2=74.9833*CNST
      D2R= 0.0
      D2F=0.0
      ADD2=2.06
      DED2=6.05
      WD= 8.11
      CC=0.81
      DEL=0.00025*25.4
C
C...  INPUT NORMINAL RADIUS OF GEAR CUTTER AND POINT WIDTH, BLADE ANGLE
C
      RU2=152.4000/2.0
      PW2=2.79
      ALP2=20.0*CNST
      DC2=2.0*RU2
C
C...  INPUT THE SYNTHESIS CONDITION PARAMETERS AND PINION CUTTER BLADE
C     ANGLE, ALP1(FOR GEAR CONVEX, ALP1>0, FOR GEAR CONCAVE ALP1<0)
C
C     **** GEAR CONVEX SIDE
C
      FI21=-0.0008
      KD=0.180
      ETAG=-65.0*CNST
      GAMA1=13.3333*CNST
      RHO= 250.0
      C2= 0.00
      ALP1=18.500*CNST
      TN1I=8.0
C...
      SGN=DSIN(ALP1)/DABS(DSIN(ALP1))
      KSIDE=0
C
C...
C
      GOTO 1989
C
C     **** GEAR CONCAVE SIDE
C
 1990 CONTINUE
      FI21= 0.0008
      KD=0.180
      ETAG= 65.0*CNST
      GAMA1=13.3333*CNST
      RHO= 200.0
      C2= 0.00
      ALP1=-21.50*CNST
      TN1I=8.0
C...
      SGN=DSIN(ALP1)/DABS(DSIN(ALP1))
      KSIDE=1
      jc1=2
```

```
C
C
C...  INPUT GEAR MACHINE TOOL SETTINGS
C
c1989 Q2= 52.6589*CNST
c     SR2=3.8872*25.4
c     XG2=0.0
c     XB2=-0.0333*25.4
c     CR2=.9772974
C     RAG=1.0/CR2
c     GAMA2=RGMA2
c     EM2=0.0
c     RC2=RU2-SGN*PW2/2.0
c     ALP2= SGN*ALP2
C
C
C...  CALCULATE GEAR MACHINE TOOL SETTINGS
C
 1989 hg=mcd*dcos(pgma2-rgma2)-ru2*dsin(b2)
      vg=ru2*dcos(b2)
      q2=datan(vg/hg)
      sr2=dsqrt(hg**2+vg**2)
      xg2=0.0
      GAMA2=RGMA2
      xb2=d2r*dsin(gama2)
      EM2=0.0
      rag=dcos(pgma2-rgma2)/dsin(pgma2)
      cr2=1.0/rag
      RC2=RU2-SGN*PW2/2.0
      ALP2= SGN*ALP2
C
C...  DELT IS THE CAM SETING
C
      DELT=0.0
C
C...  DEFINE THE MEAN CONTACT POINT
C
      V=0.000
      H=0.000
      FA=FGMA2-PGMA2
      RA=PGMA2-RGMA2
      HM=CC+WD-0.5*FW*(DTAN(FA)+DTAN(RA))
      DED2R=DED2-0.5*FW*DTAN(RA)
      XL=MCD*DCOS(PGMA2)+(DED2R-HM/2.0)*DSIN(PGMA2)
      RL=MCD*DSIN(PGMA2)-(DED2R-HM/2.0)*DCOS(PGMA2)
C...
      AGL=DATAN(RL/XL)
      OX=-DSQRT(XL**2+RL**2)*DCOS(AGL-RGMA2)
      OY=-D2R*DSIN(RGMA2)
C     WRITE(9,11) OX,OY,XL,RL
C
C
C
C...  FIND SURFACE COORDINATES OF THE MEAN CONTACT POINT
C
      ERRREL=0.1D-10
      N=2
      ITMAX=200
      IF (JCH.EQ.1) THEN
      Q2=-Q2
```

```
      XI(1)=270.0*CNST+B2
      ELSE
      XI(1)=B2
      END IF
      XI(2)=0.0
      CALL DNEQNF(FCN1,ERRREL,N,ITMAX,XI,X,FNORM)
      TH=X(1)
      PH=X(2)
      ST=DSIN(TH)
      CT=DCOS(TH)
      SH=DSIN(PH)
      CS=DCOS(PH)
      SP=DSIN(ALP2)
      CP=DCOS(ALP2)
      SM=DSIN(GAMA2)
      CM=DCOS(GAMA2)
      THIG=TH
C     WRITE(9,11) Xn2M,Yn2M,Zn2M,ZNM,YNM,ZNM
C     WRITE(9,11) XM,YM,ZM,sg,hm
C
C...  DEFINE VECTORS TO COMPUTER THE SECOND ORDER PROPERTY OF GEAR
C
      ES(1)=-DSIN(TH-PH)
      ES(2)= DCOS(TH-PH)
      ES(3)= 0.0
      EQ(1)=-SP*DCOS(TH-PH)
      EQ(2)=-SP*DSIN(TH-PH)
      EQ(3)=-CP
      CN(1)=XNM
      CN(2)=YNM
      CN(3)=ZNM
      KS=CP/(RC2-SG*SP)
      KQ=0.0
      W1(1)=-CM
      W1(2)= 0.0
      W1(3)=-SM
      W2(1)= 0.0
      W2(2)= 0.0
      W2(3)=-CR2
      VT1(1)= YM*SM+EM2*SM
      VT1(2)=-XM*SM+(ZM-XB2)*CM
      VT1(3)=-YM*CM-EM2*CM
      VT2(1)= YM*CR2
      VT2(2)=-XM*CR2
      VT2(3)= 0.0
      DO 10 I=1,3
      W12(I)=W1(I)-W2(I)
      V12(I)=VT1(I)-VT2(I)
   10 CONTINUE
C
C...  FIND THE PRINCIPAL DIRECTION AND CURVATURES AT MEAN POINT
C
      PI21=0.0
      CALL CURVA1
C     WRITE(9,12) KF,KH,SIGSF
C 12  FORMAT(3X,3(G14.7,2X))
      K2I=KF
      K2II=KH
      PHI2=PH/CR2
      sh2=dsin(phi2)
```

```
      ch2=dcos(phi2)
      xX= CM*ef(1)+SM*ef(3)
      yY= ef(2)
      zZ=-SM*ef(1)+CM*ef(3)
      ef(1)=xx
      ef(2)= CH2*yY-SH2*zZ
      ef(3)= SH2*yY+CH2*zZ
C     WRITE(9,11) xx,yy,zz
c
      xX= CM*eh(1)+SM*eh(3)
      yY= eh(2)
      zZ=-SM*eh(1)+CM*eh(3)
      eh(1)=xx
      eh(2)= CH2*yY-SH2*zZ
      eh(3)= SH2*yY+CH2*zZ
C     WRITE(9,11) Ef(1),Ef(2),Ef(3)
C     WRITE(9,11) Eh(1),Eh(2),Eh(3)
C
C
C
      ERRREL=0.1D-10
      N=1
      ITMAX=200
      XI(1)=0.0
      CALL DNEQNF(FCN2,ERRREL,N,ITMAX,XI,X,FNORM)
      PHI2PO=X(1)
C     WRITE(9,11) X(1)
C     WRITE(9,11) XH2,YH2,ZH2
C     WRITE(9,11) XNH2,YNH2,ZNH2
C
      CHP=DCOS(X(1))
      SHP=DSIN(X(1))
      CMM=DCOS(GAMMA)
      SMM=DSIN(GAMMA)
      XX= ef(1)
      YY=-ef(2)*CHP+ef(3)*shp
      ZZ=-ef(2)*SHP-ef(3)*chp
      EF(1)= XX*CMM+ZZ*SMM
      ef(2)= YY
      EF(3)=-XX*SMM+ZZ*CMM
C...
      XX= eh(1)
      YY=-eh(2)*CHP+eh(3)*shp
      ZZ=-eh(2)*SHP-eh(3)*chp
      EH(1)= XX*CMM+ZZ*SMM
      eh(2)= YY
      EH(3)=-XX*SMM+ZZ*CMM
C     WRITE(9,11) EF(1),EF(2),EF(3)
C     WRITE(9,11) EH(1),EH(2),EH(3)
      ETAG=90.0*CNST+SIGSF+ETAG
C
C     LOCAL SYNTHESIS AT MEAN CONTACT POINT
C
C...
      RH(1)=XH2
      RH(2)=YH2
      RH(3)=ZH2
      GNH(1)=XNH2
      GNH(2)=YNH2
      GNH(3)=ZNH2
```

```
      E2IH(1)= EF(1)
      E2IH(2)= EF(2)
      E2IH(3)= EF(3)
      E2IIH(1)= EH(1)
      E2IIH(2)= EH(2)
      E2IIH(3)= EH(3)
      K2I=KF
      K2II=KH
C...
C...  RELATIVE MOTION PARAMETERS IN GEAR & PINION MESHING PROCESS
C...
      R12=TN1/IN2
      W1(1)=-1.0D00
      W1(2)=0.0D00
      W1(3)=0.0D00
      W2(1)= R12*CMM
      W2(2)=0.0D00
      W2(3)=-R12*SMM
      W12(1)=W1(1)-W2(1)
      W12(2)=W1(2)-W2(2)
      W12(3)=W1(3)-W2(3)
      VT1(1)= 0.0D00
      VT1(2)= ZH2
      VT1(3)=-YH2
      VT2(1)= R12*(YH2-C)*SMM
      VT2(2)=-R12*(XH2*SMM+ZH2*CMM)
      VT2(3)= R12*(YH2-C)*CMM
      V12(1)= VT1(1)-VT2(1)
      V12(2)= VT1(2)-VT2(2)
      V12(3)= VT1(3)-VT2(3)
C     WRITE(9,3) V12(1),V12(2),V12(3)
C3    FORMAT(5X,3G14.7,/)
C...
C...  CALCULATE THE COEFFICIENT A13,A23,A33
C...
      ESN(1)= GNH(2)*E2IH(3)-GNH(3)*E2IH(2)
      ESN(2)=-(GNH(1)*E2IH(3)-GNH(3)*E2IH(1))
      ESN(3)= GNH(1)*E2IH(2)-GNH(2)*E2IH(1)
C...
      EQN(1)= GNH(2)*E2IIH(3)-GNH(3)*E2IIH(2)
      EQN(2)=-(GNH(1)*E2IIH(3)-GNH(3)*E2IIH(1))
      EQN(3)= GNH(1)*E2IIH(2)-GNH(2)*E2IIH(1)
C...
      W1VT2(1)= W1(2)*VT2(3)-W1(3)*VT2(2)
      W1VT2(2)=-(W1(1)*VT2(3)-W1(3)*VT2(1))
      W1VT2(3)= W1(1)*VT2(2)-W1(2)*VT2(1)
C...
      W2VT1(1)= W2(2)*VT1(3)-W2(3)*VT1(2)
      W2VT1(2)=-(W2(1)*VT1(3)-W2(3)*VT1(1))
      W2VT1(3)= W2(1)*VT1(2)-W2(2)*VT1(1)
C...
      WV12(1)= W12(2)*V12(3)-W12(3)*V12(2)
      WV12(2)=-(W12(1)*V12(3)-W12(3)*V12(1))
      WV12(3)= W12(1)*V12(2)-W12(2)*V12(1)
C...
      V12S=0.0D00
      V12Q=0.0D00
      WNES=0.0D00
      WNEQ=0.0D00
      VWN= 0.0D00
```

```
      W1TN=0.0D00
      W2TN=0.0D00
      VT2N=0.0D00
C...
      DO 1 I=1,3
      V12S= V12(I)*E2IH(I)+V12S
      V12Q= V12(I)*E2IIH(I)+V12Q
      WNES= W12(I)*ESN(I)+WNES
      WNEQ= W12(I)*EQN(I)+WNEQ
      VWN =GNH(I)*WV12(I)+VWN
      W1TN=GNH(I)*W1VT2(I)+W1TN
      W2TN=GNH(I)*W2VT1(I)+W2TN
      VT2N= GNH(I)*VT2(I)+VT2N
 1    CONTINUE
C     WRITE(9,6) V12S,V12Q
C 6   FORMAT(5X,2G14.7,/)
C...
C...   COMPUTER THE COEFFICIENTS A13,A23,A33
C...
      B13=-K2I*V12S-WNES
      B23=-K2II*V12Q-WNEQ
      B33=K2I*V12S**2+K2II*V12Q**2-VWN-W1TN+W2TN+VT2N*FI21*TN2/TN1
C...
C...  LOCAL SYNTHESIS OF MESHING AT MEAN CONTACT POINT
C...
      DL=KD*FW
      SIGK2= K2I+K2II
      SIGG2= K2I-K2II
      A=DEL/DL**2
      T1=-B13*V12Q+(B33+B13*V12S)*DTAN(ETAG)
      T2=B33+B23*(V12Q-V12S*DTAN(ETAG))
      ETAP=DATAN(T1/T2)
      VS1 =B33/(B13+B23*DTAN(ETAP))
      AM1=DSIN(ETAP)**2
      AM2=-DSIN(2.0D00*ETAP)/2.0D00
      AN1=(B13-B23*DTAN(ETAP))/((1.0D00+DTAN(ETAP)**2)*VS1)
      AN2=(B13*DTAN(ETAP)-B23)/((1.0D00+DTAN(ETAP)**2)*VS1)
      SGN=DSIN(ALP1)/DABS(DSIN(ALP1))
      A=A*SGN
      SIGK=(4.0D00*A**2-(AN1**2+AN2**2))/( 2.0D00*A-(AN1*DCOS(2.0D00*
     &ETAP)+AN2*DSIN(2.0D00*ETAP)))
      SIGK1= SIGK2-SIGK
      T1= 2.0D00*AN2-SIGK*DSIN(2.0D00*ETAP)
      T2=SIGG2-2.0D00*AN1-SIGK*DCOS(2.0D00*ETAP)
      SIG12=.50D00*DATAN(T1/T2)
      SIGG1=(2.0D00*AN2-SIGK*DSIN(2.0D00*ETAP))/DSIN(2.0D00*SIG12)
      K1I=(SIGK1+SIGG1)/2.0D00
      K1II=(SIGK1-SIGG1)/2.0D00
C..
C..
C..
C     WRITE(9,11) ETAP,K1I,K1II
C     WRITE(9,11) SIGK,SIG12,SIGK1,SIGG1
C     WRITE(9,8) T1,T2
C  8  FORMAT(5X,3G14.7)
C..
C..
C...
C...  PRINCIPLE DIRECTIONS OF PINION SURFACE AT POINT M
C...
```

```
      DO 15 I=1,3
      E1IH(I)= DCOS(SIG12)*E2IH(I)-DSIN(SIG12)*E2IIH(I)
      E1IIH(I)= DSIN(SIG12)*E2IH(I)+DCOS(SIG12)*E2IIH(I)
   15 CONTINUE
C     WRITE(9,11) E1IH(1),E1IH(2),E1IH(3)
C     WRITE(9,11) E1IIH(1),E1IIH(2),E1IIH(3)
C...
C...  COINCIDE THE NORMALS OF CUTTER AND THE PINION SURFACES
C...
      SM1=DSIN(GAMA1)
      CM1=DCOS(GAMA1)
      SP=DSIN(ALP1)
      CP=DCOS(ALP1)
      T1=-(XNH2+SP*SM1)
      T2= CP*CM1
      IF (JCH.EQ.1) THEN
      THF=DARCOS(T1/T2)
      ELSE
      THF=DARCOS(T1/T2)
      THF=360.0*CNST-THF
      END IF
      BA1=-CP*DSIN(THF)
      BA2= CP*DSIN(GAMA1)*DCOS(THF)-SP*DCOS(GAMA1)
      TT=-(YNH2**2+ZNH2**2)
      CSH=-(BA1*YNH2+BA2*ZNH2)/TT
      SNH=(BA2*YNH2-BA1*ZNH2)/TT
      PHIH=2.0*DATAN2(SNH,(1.0D00+CSH))
C     WRITE(9,8) THF,PHIH
C
C...  FIND THE PRINCIPAL DIRECTIONS OF PINION GENERATING SURFACE
C
      EFI(1)=-DSIN(THF)
      EFI(2)= DCOS(THF)
      EFI(3)= 0.0D00
      EFII(1)= SP*DCOS(THF)
      EFII(2)= SP*DSIN(THF)
      EFII(3)=-CP
C     WRITE(9,11) EFI(1),EFI(2),EFI(3)
C     WRITE(9,11) EFII(1),EFII(2),EFII(3)
C
C...  FIND THE PINION PRINCIPAL DIRECTIONS IN SYSTEM SM1
C
      XX= E1IH(1)
      YY= DCOS(PHIH)*E1IH(2)+DSIN(PHIH)*E1IH(3)
      ZZ=-DSIN(PHIH)*E1IH(2)+DCOS(PHIH)*E1IH(3)
      E1I(1)= CM1*XX-SM1*ZZ
      E1I(2)= YY
      E1I(3)= SM1*XX+CM1*ZZ
C
      XX= E1IIH(1)
      YY= DCOS(PHIH)*E1IIH(2)+DSIN(PHIH)*E1IIH(3)
      ZZ=-DSIN(PHIH)*E1IIH(2)+DCOS(PHIH)*E1IIH(3)
      E1II(1)= CM1*XX-SM1*ZZ
      E1II(2)= YY
      E1II(3)= SM1*XX+CM1*ZZ
C
C...  FIND THE UNIT NORNMAL IN SYSTEM SM1
C
      XX= XNH2
      YY= DCOS(PHIH)*YNH2+DSIN(PHIH)*ZNH2
```

```
      ZZ=-DSIN(PHIH)*YNH2+DCOS(PHIH)*ZNH2
      GN(1)= CM1*XX-SM1*ZZ
      GN(2)= YY
      GN(3)= SM1*XX+CM1*ZZ
C
C...  EXPRESS THE POSITION VECTOR IN SM1
C
      XX= XH2
      YY= DCOS(PHIH)*YH2+DSIN(PHIH)*ZH2
      ZZ=-DSIN(PHIH)*YH2-DCOS(PHIH)*ZH2
      RX= CM1*XX-SM1*ZZ
      RY= YY
      RZ= SM1*XX+CM1*ZZ
C     XX=-CP*DCOS(THF)
C     YY=-CP*DSIN(THF)
C     ZZ=-SP
C...
C     WRITE(9,11) E1I(1),E1I(2),E1I(3)
C     WRITE(9,11) E1II(1),E1II(2),E1II(3)
C     WRITE(9,11) GN(1),GN(2),GN(3)
C     WRITE(9,11) XX,YY,ZZ
C     WRITE(9,11) RX,RY,RZ
C...
      DO 20 I=1,3
      XX=-EFI(I)
      EFI(I)= EFII(I)
      EFII(I)=XX
 20   CONTINUE
C
C     WRITE(9,11) E1I(1),E1I(2),E1I(3)
C     WRITE(9,11) E1II(1),E1II(2),E1II(3)
C...
C...  FIND THE ANGLE FORMED BETWEEN PRINCIPAL CURVATURES
C...
      EFE1(1)= EFI(2)*E1I(3)-EFI(3)*E1I(2)
      EFE1(2)=-EFI(1)*E1I(3)-EFI(3)*E1I(1)
      EFE1(3)= EFI(1)*E1I(2)-EFI(2)*E1I(1)
      T1=0.0D00
      T2=0.0D00
      DO 30 I=1,3
      T2=EFI(I)*E1I(I)+T2
      T1=GN(I)*EFE1(I)+T1
   30 CONTINUE
      SIGF1=2.0*DATAN2(T1,1.0+T2)
C
C...  FIND THE CURVATURE OF PINION GENERATION SURFACE AT MEAN POINT
C
      IF (JCC.EQ.1) THEN
      KFI=0.0
      B12=0.5D00*(K1I-K1II)*DSIN(-2.0D00*SIGF1)
      B11=KFI-K1I*DCOS(SIGF1)**2-K1II*DSIN(SIGF1)**2
      TKK= K1I*DSIN(SIGF1)**2+K1II*DCOS(SIGF1)**2
      KFII=(B12**2+B11*TKK)/B11
C     WRITE(9,11) SIGF1,KFII
C
C...  FIND THE CUTTER POINT RADIUS AND ITS CENTER
C
C     SF=SG*DCOS(ALP2)/DCOS(ALP1)
      SF=DABS(RZ)/DCOS(ALP1)
      RCF=CP/DABS(KFII)-SF*SP
```

```
      DO 40 I=1,3
      P(I)= GN(I)*CP-EFI(I)*SP
 40   CONTINUE
      RCX= RX-SF*EFI(1)+RCF*P(1)
      RCY= RY-SF*EFI(2)+RCF*P(2)
      RCZ= RZ-SF*EFI(3)+RCF*P(3)
C     WRITE(9,11) RCF,SF
C     WRITE(9,11) RCX,RCY,RCZ
      ELSE
      KFI=1.0/RHO
      B12=0.5D00*(K1I-K1II)*DSIN(-2.0D00*SIGF1)
      B11=KFI-K1I*DCOS(SIGF1)**2-K1II*DSIN(SIGF1)**2
      TKK= K1I*DSIN(SIGF1)**2+K1II*DCOS(SIGF1)**2
      KFII=(B12**2+B11*TKK)/B11
C     WRITE(9,11) SIGF1,KFII
      DBT=-RZ
      RM=CP/DABS(KFII)
      ZO=-(DBT+RHO*SP)
      XO=RM-RHO*CP
      RCF=XO+RHO*DSQRT(1.0-(ZO/RHO)**2)
      RCX=RHO*GN(1)-XO*DCOS(THF)+RX
      RCY=RHO*GN(2)-XO*DSIN(THF)+RY
      RCZ=RHO*GN(3)-ZO+RZ
C     WRITE(9,777) XO,ZO
C777  FORMAT(3X,' XO, XO =',2(2X,G14.7))
C     WRITE(9,11) RCF,RM,DBT
C     WRITE(9,11) RCX,RCY,RCZ
      END IF
C
C
C...  THE FOLLOWING IS TO FIND THE CUTTING RATIO
C
      CSM1=CM1
      SNM1=SM1
      T1X=EFI(1)
      T1Y=EFI(2)
      T1Z=EFI(3)
      T2X=EFII(1)
      T2Y=EFII(2)
      T2Z=EFII(3)
      XN=GN(1)
      YN=GN(2)
      ZN=GN(3)
      RXC=RX
      RYC=RY
      RZC=RZ
C
C      WRITE(9,11) RCX,RCY,RCZ
C      WRITE(9,11) T1X,T1Y,T1Z
C      WRITE(9,11) T2X,T2Y,T2Z
C      WRITE(9,11) XN,YN,ZN
C      WRITE(9,11) RXC,RYC,RZC
C...
C...  THE FOLLOWING IS TO DETERMINE DELTA,EM,AND IFM
C...
      M11=XN*T1Y-YN*T1X
      M12=-CSM1*(YN*T1Z-ZN*T1Y)
      M21=XN*T2Y-YN*T2X
      M22=-CSM1*(YN*T2Z-ZN*T2Y)
C     WRITE(9,11) M11,M12,M21,M22
```

```
C      L11=(B12/B11*M11-M21)/KFII
C      L12=(B12/B11*M12-M22)/KFII
C...
C      L21=-T2Z/T1Z*L11
C      L22=-T2Z/T1Z*L12-RYC*CSM1/T1Z
       DTT=B12*KFI*T2Z+B11*KFII*T1Z
       L11=-T1Z*(B11*M21-B12*M11)/DTT
       L12=(-B12*KFI*RYC*CSM1-T1Z*(B11*M22-B12*M12))/DTT
       L21=T2Z*(B11*M21-B12*M11)/DTT
       L22=(-B11*KFII*RYC*CSM1+T2Z*(B11*M22-B12*M12))/DTT
C      WRITE(9,11) L11,L12,L21,L22
       X11=L21*T1X+L11*T2X
       X12=L22*T1X+L12*T2X
       X21=L21*T1Y+L11*T2Y
       X22=L22*T1Y+L12*T2Y
       X31=L21*T1Z+L11*T2Z
       X32=L22*T1Z+L12*T2Z
       E11=YN*X11-XN*X21
       E12=YN*X12-XN*X22-ZN*X21*CSM1-YN*X31*CSM1
       E13=-(ZN*X22-YN*X32)*CSM1
       Y11=-XN*(RXC*SNM1-RZC*CSM1)-YN*RYC*SNM1
       Y12=Y11*SNM1
       X13=X12-RYC*SNM1
       X23=X22+RXC*SNM1-RZC*CSM1
       X33=X32+RYC*CSM1
       Y21=-XN*X21*SNM1+ZN*X21*CSM1-YN*(X11*SNM1-X31*CSM1)
       Y22=-XN*X23*SNM1+YN*(SNM1*X13-CSM1*X33)-ZN*X23*CSM1
C
C...
C
C...
C...   THE EFFECT OF SECOND ORDER RATIO OF ROLL ON A33
       TM1=XN*X11-YN*X21-ZN*X31
       TM2=XN*X13+YN*X23+ZN*X33
C      WRITE(9,163) TM1,TM2
C163   FORMAT(2X,'TM1,TM2 ',2(2X,G14.7))
       ZZ1=C2*TM1
       ZZ2=C2*(TM2+SNM1*TM1)
       ZZ3=C2*SNM1*TM2
C...
C      Z1=KFII*L11**2-E11
C      Z2=2.0D00*KFII*L11*L12-E12-Y21-Y11
C      Z3=KFII*L12**2-E13-Y22+Y12
       Z1=KFI*L21**2+KFII*L11**2-E11-ZZ1
       Z2=2.0*KFI*L21*L22+2.0D00*KFII*L11*L12-E12-Y21-Y11-ZZ2
       Z3=KFI*L22**2+KFII*L12**2-E13-Y22+Y12+ZZ3
       N11=KFII*L11+M21
       N12=KFII*L12+M22
       N21=KFI*L21+M11
       N22=KFI*L22+M12
C      WRITE(9,11) B12,Z1,M11,N11
       AA=B12*Z1-N21*N11
       BB=B12*Z2-N21*N12-N22*N11
       CCC=B12*Z3-N22*N12
C      WRITE (9,11) AA,BB,CCC
       IF (AA.GT.0.000001) GOTO 1949
       T1=-CCC/BB
       GOTO 1950
 1949  T1=(-BB+DSQRT(BB**2-4.0D00*AA*CCC))/(2.0D00*AA)
 1950  FM1=T1+SNM1
```

```
      CR1=FM1
      RAP=1.0/CR1
      VF3=X31*T1+X32+RYC*CSM1
C...
C...
C...
C...  THE DETERMINATION OF EM AND DELTA
C...
      EM1=(X11*T1+RYC*FM1+X13)/FM1
      XG1=(X21*T1-RXC*FM1+X23)/(FM1*CSM1)
C...
      RCX=RCX+XG1*CSM1
      RCY=RCY-EM1
      RCZ=RCZ+XG1*SNM1
C...
      V1=RCY
      H1=RCX
      XB1=RCZ
      SR1=DSQRT(V1**2+H1**2)
      Q1=-DARSIN(V1/SR1)
      XB1=-XB1
C
C.... DETERMINE THE CAM SETTING
C
      RA1=1.0/CR1
      RAM=TN1/TN1I*RA1
      PSI1=DATAN(C2*RAM/(RAM-1.0))
      RUP=15.0*DCOS(PSI1)/(RAM-1.0)
      RU1=RUP
C     DELT=0.0
C
      CALL CAM
C     WRITE(9,191) RUP
C     WRITE(9,191) RU1
C191  FORMAT(2X,'RU1 DELT = ',2(2X,G14.7))
C     WRITE(9,199) RA1,C2,D6,E24,F120
C     WRITE(9,199) RA1,CPF,DPF,EPF,FPF
C 199 FORMAT(2X,'CAM',2X,5(2X,G14.7))
C
C...
C
C     WRITE(9,25) FM1
C 25  FORMAT(5X,'  FM1 = ',G14.7)
C     WRITE(9,44) EM1,SR1,Q1
C44   FORMAT(2X,'EM1,SR1,Q1',3(3X,G14.7))
C     WRITE(9,45) XG1,XB1,V1,H1
C45   FORMAT(2X,'XG1,XB1,V1,H1',4(2X,G14.7))
C...
      IF (KSIDE.EQ.0.0) THEN
      WRITE(9,131)
 131  FORMAT(/2X,'******************************************',/
     $        2X,'* OUTPUT FOR GEAR CONVEX SIDE           *',/
     &        2X,'******************************************',/)
      ELSE
      WRITE(9,331)
 331  FORMAT(/2X,'******************************************',/
     $        2X,'* OUTPUT FOR GEAR CONCAVE SIDE          *',/
     &        2X,'******************************************',/)
      END IF
      WRITE(9,13)
```

```
 13   FORMAT(/2X,'******************************************',/
    $         2X,'*   GEAR CUTTER SPICIFICATIONS            *',/
    &         2X,'******************************************',/)
      WRITE(9,115) DC2,PW2,ALP2
 115  FORMAT(/2X,' GEAR CUTTER DIAMETER :   DC2 =' ,G14.7,/
    $         2X,' CUTTER POINT WIDTH :      PW =' ,G14.7,/
    &         2X,' CUTTER BLADE ANGLE :    PHI2 =' ,G14.7,//)
      WRITE(9,3)
 3    FORMAT(/2X,'******************************************',/
    $         2X,'*   BASIC GEAR MACHINE-TOOL SETTINGS     *',/
    &         2X,'******************************************',/)
      WRITE(9,4) Q2,SR2,XG2,XB2,EM2,GAMA2,RAG
 4    FORMAT(/2X,' BASIC CRADLE ANGLE :        Q2 =' ,G14.7,/
    &         2X,' RADIAL SETTING :           SR2 =' ,G14.7,/
    #         2X,' MACHINE CENTER TO BACK : XG2 =' ,G14.7,/
    #         2X,' SLIDING BASE :             XB2 =' ,G14.7,/
    $         2X,' BLANK OFFSET :             EM2 =' ,G14.7,/
    #         2X,' MACHINE ROOT ANGLE :   GAMA2 =' ,G14.7,/
    #         2X,' RATIO OF ROLL :            RAG =' ,G14.7,//)
      WRITE(9,6)
 6    FORMAT(//2X,'******************************************',/
    &          2X,'*   BASIC PINION MACHINE-TOOL SETTINGS   *',/
    &          2X,'******************************************',/)
      WRITE(9,7) ALP1, RCF, Q1,SR1,XG1,XB1,EM1,GAMA1,RAP
 7    FORMAT(/2X,' BLADE ANGLE :              ALP1 =' ,G14.7,/
    &         2X,' POINT RADIUS :              RCF =' ,G14.7,/
    &         2X,' BASIC CRADLE ANGLE :         Q1 =' ,G14.7,/
    $         2X,' RADIAL SETTING :            SR1 =' ,G14.7,/
    &         2X,' MACHINE CENTER TO BACK : XG1 =' ,G14.7,/
    $         2X,' SLIDING BASE :              XB1 =' ,G14.7,/
    &         2X,' BLANK OFFSET :              EM1 =' ,G14.7,/
    $         2X,' MACHINE ROOT ANGLE :    GAMA1 =' ,G14.7,/
    &         2X,' RATIO OF ROLL :             RAP =' ,G14.7,//)
      IF (JCC.EQ.2) THEN
      WRITE(9,61)
 61   FORMAT(//2X,'******************************************',/
    &          2X,'*   COORDINATES OF THE CENTER OF THE ARC *',/
    &          2X,'******************************************',/
      WRITE(9,71) XO,ZO
 71   FORMAT(/2X,' RADIAL COORDINATE :         XO =' ,G14.7,/
    &         2X,' AXIAL COORDINATE :          ZO =' ,G14.7,//)
      ELSE
      GOTO 1919
      END IF
1919  CONTINUE
      WRITE(9,16)
 16   FORMAT(//2X,'************************************************',/
    &          2X,'*CAM SETTINGS AND COEFFICIENTS OF TAYLOR SERIES*',/
    &          2X,'************************************************',/)
      WRITE(9,17) PSI1, RUP, DELT, RA1,C2,D6,E24,F120
 17   FORMAT(/2X,' GUIDE ANGLE :               PSI1 =' ,G14.7,/
    &         2X,' CAM PITCH RADIUS :          RUP =' ,G14.7,/
    &         2X,' CAM SETTING :              DELT =' ,G14.7,/
    &         2X,' 1ST ORDER COEFFICIENT :   RA1 =' ,G14.7,/
    &         2X,' 2ND ORDER COEFFICIENT :    C2 =' ,G14.7,/
    &         2X,' 3RD ORDER COEFFICIENT :    D6 =' ,G14.7,/
    &         2X,' 4TH ORDER COEFFICIENT :   E24 =' ,G14.7,/
    &         2X,' 5TH ORDER COEFFICIENT : F120 =' ,G14.7,//)
C
C...  CALL TCA
```

```
C
C...
C...  DEFINE THE INITIAL POINT
C...
C...
      XI(1)=THIG
      XI(2)= 0.000000
      XI(3)=THF
      XI(4)=0.0
      XI(5)= 0.00
C
C...  FIND THE INITIAL CONTACT POINT
C
 5555 N=5
      ERRREL=0.1D-10
      ITMAX=200
      PHI2P=PHI2P0
      IF (JCC.EQ.1) THEN
      CALL DNEQNF(FCN,ERRREL,N,ITMAX,XI,X,FNORM)
      ELSE
      CALL DNEQNF(FCNR,ERRREL,N,ITMAX,XI,X,FNORM)
      END IF
      PHI1P0=X(5)
C
C
C
C
      PHI2P1=PHI2P0-180.0*CNST/TN2-TL1*180.0*CNST/(6.0*TN2)
      PHI2P2=PHI2P0+180.0*CNST/TN2+TL2*180.0*CNST/(6.0*TN2)
      KK=1
      PHI2P=PHI2P1
  333 CONTINUE
      IF (JCC.EQ.1) THEN
      CALL DNEQNF(FCN,ERRREL,N,ITMAX,XI,X,FNORM)
      ELSE
      CALL DNEQNF(FCNR,ERRREL,N,ITMAX,XI,X,FNORM)
      END IF
C     XI(1)=X(1)
C     XI(2)=X(2)
C     XI(3)=X(3)
C     XI(4)=X(4)
C     XI(5)=X(5)
C
C...  find the transmission error
C
C      ERRR=PHI2P-PHI2P0-TN1/TN2*(X(5)-PHI1P0)
       ERRR=PHI2P-PHI2P0+TN1/TN2*(X(5)-PHI1P0)
      ERR(KK)=3600.0*ERRR/CNST
      PI2P(KK)=PHI2P
C
C...  computer the contact path
C
      x1c= x2m
      r1c= dsqrt(y2m**2+z2m**2)
      xcp(KK)= x1c*dcos(rgma2)+r1c*dsin(rgma2)+ox
      ycp(KK)=-x1c*dsin(rgma2)+r1c*dcos(rgma2)+oy
C
C...  COMPUTER THE PRINCIPAL DIRECTIONS AND CURVATURES OF GEAR
C
      TH=X(1)
```

```
      PH=X(2)
      ST=DSIN(TH)
      CT=DCOS(TH)
      SH=DSIN(PH)
      CS=DCOS(PH)
      SP=DSIN(ALP2)
      CP=DCOS(ALP2)
      SM=DSIN(GAMA2)
      CM=DCOS(GAMA2)
C
C...  DEFINE VECTORS TO COMPUTER THE SECOND ORDER PROPERTY OF GEAR
C
      ES(1)=-DSIN(TH-PH)
      ES(2)= DCOS(TH-PH)
      ES(3)= 0.0
      EQ(1)=-SP*DCOS(TH-PH)
      EQ(2)=-SP*DSIN(TH-PH)
      EQ(3)=-CP
      CN(1)=XNM
      CN(2)=YNM
      CN(3)=ZNM
      KS=CP/(RC2-SG*SP)
      KQ=0.0
      W1(1)=-CM
      W1(2)= 0.0
      W1(3)=-SM
      W2(1)= 0.0
      W2(2)= 0.0
      W2(3)=-CR2
      VT1(1)= YM*SM+EM2*SM
      VT1(2)=-XM*SM+(ZM-XB2)*CM
      VT1(3)=-YM*CM-EM2*CM
      VT2(1)= YM*CR2
      VT2(2)=-XM*CR2
      VT2(3)= 0.0
      DO 110 I=1,3
      W12(I)=W1(I)-W2(I)
      V12(I)=VT1(I)-VT2(I)
  110 CONTINUE
C
C
      PI21=0.0
      CALL CURVA1
      K2I=KF
      K2II=KH
      PHI2=PH/CR2
      sh2=dsin(phi2)
      ch2=dcos(phi2)
      xX= CM*ef(1)+SM*ef(3)
      yY= ef(2)
      zZ=-SM*ef(1)+CM*ef(3)
      ef(1)=xx
      ef(2)= CH2*yY-SH2*zZ
      ef(3)= SH2*yY+CH2*zZ
c
      xX= CM*eh(1)+SM*eh(3)
      yY= eh(2)
      zZ=-SM*eh(1)+CM*eh(3)
      eh(1) xx
      eh(2)= CH2*yY-SH2*zZ
```

```
      eh(3)= SH2*yY+CH2*zZ
C...
      CHP=DCOS(PHI2P)
      SHP=DSIN(PHI2P)
      CMM=DCOS(GAMMA)
      SMM=DSIN(GAMMA)
      XX= ef(1)
      YY=-ef(2)*CHP+ef(3)*shp
      ZZ=-ef(2)*SHP-ef(3)*chp
      E2IH(1)= XX*CMM+ZZ*SMM
      E2IH(2)= YY
      E2IH(3)=-XX*SMM+ZZ*CMM
C...
      XX= eh(1)
      YY=-eh(2)*CHP+eh(3)*shp
      ZZ=-eh(2)*SHP-eh(3)*chp
      E2IIH(1)= XX*CMM+ZZ*SMM
      E2IIH(2)= YY
      E2IIH(3)=-XX*SMM+ZZ*CMM
C
C...  COMPUTER THE PRINCIPAL DIRECTIONS AND CURVATURES OF PINION
C
      TH1=X(3)
      PH1=X(4)
      STP=DSIN(TH1+PH1)
      CTP=DCOS(TH1+PH1)
      IF(JCC.EQ.1) THEN
      SP1=DSIN(ALP1)
      CP1=DCOS(ALP1)
      ELSE
      SGN=ALP1/DABS(ALP1)
      ALP=SGN*ALP
      SP1=DSIN(ALP)
      CP1=DCOS(ALP)
      END IF
      SM1=DSIN(GAMA1)
      CM1=DCOS(GAMA1)
C
C...  DEFINE VECTORS TO COMPUTER THE SECOND ORDER PROPERTY OF PINION
C
      ES(1)=-STP
      ES(2)= CTP
      ES(3)= 0.0
      EQ(1)= SP1*CTP
      EQ(2)= SP1*STP
      EQ(3)=-CP1
      CN(1)=XNM1
      CN(2)=YNM1
      CN(3)=ZNM1
      IF (JCC.EQ.1) THEN
      KS=CP1/(RCF+SF*SP1)
      KQ=0.0
      ELSE
      KS=DCOS(ALP)/(RHO*DCOS(ALP)+XO)
      KQ=1.0/RHO
      END IF
      W1(1)= CM1
      W1(2)= 0.0
      W1(3)= SM1
      W2(1)= 0.0
```

```
      W2(2)= 0.0
      W2(3)= CR1T
      VT1(1)=-YM1*SM1-EM1*SM1
      VT1(2)= XM1*SM1-(ZM1-XB1)*CM1
      VT1(3)= YM1*CM1+EM1*CM1
      VT2(1)=-YM1*CR1T
      VT2(2)= XM1*CR1T
      VT2(3)= 0.0
      DO 210 I=1,3
      W12(I)=W1(I)-W2(I)
      V12(I)=VT1(I)-VT2(I)
  210  CONTINUE
C
C
      PI21=PCR1T
      CALL CURVA1
C     WRITE(9,12) KF,KH,SIGSF
      K1I=KF
      K1II=KH
C     PHI1=PH1/CR1
      SH1=DSIN(PHI1)
      CH1=DCOS(PHI1)
      XX= CM1*EF(1)+SM1*EF(3)
      yY= ef(2)
      ZZ=-SM1*EF(1)+CM1*EF(3)
      ef(1)=xx
      EF(2)= CH1*YY+SH1*ZZ
      EF(3)=-SH1*YY+CH1*ZZ
c
      XX= CM1*EH(1)+SM1*EH(3)
      yY= eh(2)
      ZZ=-SM1*EH(1)+CM1*EH(3)
      eh(1)=xx
      EH(2)= CH1*YY+SH1*ZZ
      EH(3)=-SH1*YY+CH1*ZZ
C...
      CH1P=DCOS(X(5))
      SH1P=DSIN(X(5))
      E1IH(1)=EF(1)
      E1IH(2)= CH1P*EF(2)-SH1P*EF(3)
      E1IH(3)= SH1P*EF(2)+CH1P*EF(3)
      E1IIH(1)=EH(1)
      E1IIH(2)= CH1P*EH(2)-SH1P*EH(3)
      E1IIH(3)= SH1P*EH(2)+CH1P*EH(3)
      DO 109 I=1,3
      E1IH(I)=-E1IH(I)
      E1IIH(I)=-E1IIH(I)
 109  CONTINUE
C
C...  COMPUTER THE DIMENSION AND ORIENTATION OF THE CONTACT ELLIPSE
C
      GNH(1)=XNH2
      GNH(2)=YNH2
      GNH(3)=ZNH2
      CALL ELLIP
      AX1(KK)=A2L
      AX2(KK)=B2L
      ANG1(KK)=TAU1R
      ANG2(KK)=TAU2R
C
```

```
      KK=KK+1
      PHI2P=PHI2P+180.0*CNST/(TN2*6.0)
      IF(PHI2P.LE.(PHI2P2+0.0001)) GOTO 333
C...
C...
      WRITE(9,441)
441   FORMAT(/,'*****************************************************',
     &        /,'*  TRANSMISSION ERROR IN A MESHING PERIOD          *',
     $        /,'*****************************************************',/)
C
      DO 444 I=1,KK-1
      PI2P(I)=PI2P(I)/CNST
      WRITE(9,555) PI2P(I),ERR(I)
555   FORMAT(3X,3(G14.7,3X))
444   CONTINUE
C
      WRITE(9,551)
551   FORMAT(/,'*****************************************************',
     &        /,'*  CONTACT PATH FOR A PAIR OF TEETH IN MESH        *',
     $        /,'*****************************************************',/)
      DO 666 I=1,KK-1
      WRITE(9,747) XCP(I),YCP(I)
 747   FORMAT(3X,2(G14.7,3X))
 666   CONTINUE
C
      WRITE(9,661)
661   FORMAT(/,'*****************************************************',
     &        /,'*  DIMENSION AND ORIENTATION OF CONTACT ELLIPSE    *',
     $        /,'*****************************************************',/)
      DO 888 I=1,KK-1
      WRITE(9,889) AX1(I),ANG1(I),AX2(I),ANG2(I)
 889   FORMAT(3X,4(G14.7,3X))
 888   CONTINUE
C
C
C
       IF(JCL.EQ.1) GOTO 1111
       IF(JCL.EQ.3) GOTO 1113
C
C...   V AND H CHECK FOR TOE POSITION
C
      HMT=WD+CC-3.0/4.0*FW*(DTAN(FA)-DTAN(RA))
      DED2T=DED2-3.0/4.0*FW*DTAN(RA)
      TMCD=MCD-0.25*FW
      XL=TMCD*DCOS(PGMA2)+(DED2T-HMT/2.0)*DSIN(PGMA2)
      RL=TMCD*DSIN(PGMA2)-(DED2T-HMT/2.0)*DCOS(PGMA2)
C
C...  FIND THE MEAN CONTACT POINT ON THE GEAR SURFACE
C
      ERRREL=0.1D-7
      N=2
      ITMAX=200
      IF (JCH.EQ.1) THEN
      XI(1)=270.0*CNST+B2
      ELSE
      XI(1)=B2
C     XI(1)=90.0*CNST-B2
      END IF
      XI(2)=0.0
      CALL DNEQNF(FCN1,ERRREL,N,ITMAX,XI,X,FNORM)
```

```
      TH=X(1)
      PH=X(2)
      ZY1=X(1)
      ZY2=X(2)
      N=3
      ERRREL=0.1D-10
      ITMAX=200
      XI(1)=0.0
      XI(2)=THF
      XI(3)= 0.0
      IF (JCC.EQ.1) THEN
      CALL DNEQNF(FCNM,ERRREL,N,ITMAX,XI,X,FNORM)
      ELSE
      CALL DNEQNF(FCNMR,ERRREL,N,ITMAX,XI,X,FNORM)
      END IF
      PHI2PO=X(1)
      XI(1)=ZY1
      XI(2)=ZY2
      XI(3)=X(2)
      XI(4)=X(3)
      XI(5)=PHI1P
      WRITE(9,149)
 149  FORMAT(//6X,'*************************************************',/
     &          6X,'*       V AND H CHECK AT TOE POSITION          *',/
     &          6X,'*************************************************',/)
      WRITE(9,139) V,H
 139  FORMAT(//4X,'*** V = ',G14.7,'*** H =',G14.7//)
C...
      JCL=3
      GO TO 5555
C
C...   V AND H CHECK FOR HEEL POSITION
C
C1113 HMH=WD-CCC-1.0/4.0*FW*(DTAN(FA)+DTAN(RA))
C     DED2H=DED2-1.0/4.0*FW*DTAN(RA)
C     HMCD=MCD+0.25*FW
 1113 HMH=WD-CC-0.16*FW*(DTAN(FA)+DTAN(RA))
      DED2H=DED2-0.16*FW*DTAN(RA)
      HMCD=MCD+0.16*FW
      XL=HMCD*DCOS(PGMA2)+(DED2H-HMH/2.0)*DSIN(PGMA2)
      RL=HMCD*DSIN(PGMA2)-(DED2H-HMH/2.0)*DCOS(PGMA2)
      ERRREL=0.1D-7
      N=2
      ITMAX=200
      IF (JCH.EQ.1) THEN
      XI(1)=270.0*CNST+B2
      ELSE
C     XI(1)=90.0*CNST-B2
      XI(1)=B2
      END IF
      XI(2)=0.0
      CALL DNEQNF(FCN1,ERRREL,N,ITMAX,XI,X,FNORM)
      TH=X(1)
      PH=X(2)
      ZY1=X(1)
      ZY2=X(2)
C...  FIND THE V AND H VALUE FOR HEEL POSITION
      N=3
      ERRREL=0.1D-10
      ITMAX=200
```

```
      XI(1)=0.00
      XI(2)=THF
      XI(3)= 0.0
C     XI(2)=THF+0.2
C     XI(3)=-0.2
      IF (JCC.EQ.1) THEN
      CALL DNEQNF(FCNM,ERRREL,N,ITMAX,XI,X,FNORM)
      ELSE
      CALL DNEQNF(FCNMR,ERRREL,N,ITMAX,XI,X,FNORM)
      END IF
      PHI2P0=X(1)
      XI(1)=ZY1
      XI(2)=ZY2
      XI(3)=X(2)
      XI(4)=X(3)
      XI(5)=PHI1P
C     WRITE(9,11) PHI2P0,PHI1P
      WRITE(9,159)
 159  FORMAT(//6X,'**************************************************',
     &       6X,'*       V AND H CHECK AT HEEL POSITION          *',
     &       6X,'**************************************************'/)
      WRITE(9,169) V,H
 169  FORMAT(//4X,'*** V = ',G14.7,'*** H =',G14.7/)
C...
      JCL=1
      GOTO 5555
 1111 CONTINUE
      IF(KSIDE.EQ.0) GOTO 1990
      STOP
      END
C
C...  FCN1 IS TO FIND THE MEAN CONTACT POINT
C
      SUBROUTINE FCN1(X,F,N)
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER N
      REAL*8 X(N),F(N),mcd
      COMMON/A1/CNST,TN1,TN2,C,FW,GAMMA,x1,r1,mcd
      COMMON/A3/B2,RGMA2,FGMA2,PGMA2,D2R,D2F,ADD2,DED2,WD,CC,D2P
      COMMON/A4/SR2,Q2,RC2,PW2,XB2,XG2,EM2,GaMA2,CR2,ALP2,PHI2,PHI2P
      COMMON/A5/SG,XM,YM,ZM,XNM,YNM,ZNM,X2M,Y2M,Z2M,XN2M,YN2M,ZN2M,
     &XNH2,YNH2,ZNH2,XH2,YH2,ZH2
      TH=X(1)
      PH=X(2)
      SP=DSIN(ALP2)
      CP=DCOS(ALP2)
      SM=DSIN(GAMA2)
      CM=DCOS(GAMA2)
      STP=DSIN(TH-PH)
      CTP=DCOS(TH-PH)
      XNM=-CP*CTP
      YNM=-CP*STP
      ZNM= SP
      AA1=RC2*STP+SR2*DSIN(-Q2-PH)
      AA2=RC2*CTP+SR2*DCOS(-Q2-PH)
      AX=-EM2*SM
      AY= XB2*CM
      AZ= EM2*CM
C
C...  FIND SG
```

```
C
      T1= XNM*(AX-AA1*(SM-CR2))+YNM*(AY+AA2*(SM-CR2))+ZNM*(AZ+AA1*CM)
      T2=-XNM*(SM-CR2)*SP*STP+YNM*((SM-CR2)*SP*CTP-CP*CM)+ZNM*CM*SP*STP
      SG=T1/T2
      XM= (RC2-SG*SP)*CTP+SR2*DCOS(-Q2-PH)
      YM= (RC2-SG*SP)*STP+SR2*DSIN(-Q2-PH)
      ZM=-SG*CP
C     XM=-SG*SP*CTP+AA2
C     YM=-SG*SP*STP+AA1
C     ZM=-SG*CP
      xX= CM*XM+SM*ZM-XG2-XB2*SM
      yY= YM+EM2
      zZ=-SM*XM+CM*ZM-XB2*CM
      XN= CM*XNM+SM*ZNM
      YN= YNM
      ZN=-SM*XNM+CM*ZNM
      PHI2=PH/CR2
      sh2=dsin(phi2)
      ch2=dcos(phi2)
      X2M= xX
      Y2M= CH2*yY-SH2*zZ
      Z2M= SH2*yY+CH2*zZ
      XN2M= XN
      YN2M= CH2*YN-SH2*ZN
      ZN2M= SH2*YN+CH2*ZN
      F(1)=X2M-XL
      F(2)=Yy**2+Zz**2-RL**2
c     F(2)=Y2M**2+Z2M**2-RL**2
      RETURN
      END
C...
C...  SUBROUTINE CURVA1 IS TO COMPUTER THE CURVATURE OF THE
C...                GENERATED SURFAFE
C...
      SUBROUTINE CURVA1
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 KS,KQ,KF,KH
      DIMENSION ESN(3),EQN(3),W1VT2(3),WV12(3),W2VT1(3)
      COMMON/A6/ES(3),EQ(3),CN(3),W1(3),W2(3),W12(3),VT1(3),VT2(3),
     $V12(3),KS,KQ,KF,KH,EF(3),EH(3),SIGSF,PI21
C...
      ESN(1)= CN(2)*ES(3)-CN(3)*ES(2)
      ESN(2)=-(CN(1)*ES(3)-CN(3)*ES(1))
      ESN(3)= CN(1)*ES(2)-CN(2)*ES(1)
C...
      EQN(1)= CN(2)*EQ(3)-CN(3)*EQ(2)
      EQN(2)=-(CN(1)*EQ(3)-CN(3)*EQ(1))
      EQN(3)= CN(1)*EQ(2)-CN(2)*EQ(1)
C...
      W1VT2(1)= W1(2)*VT2(3)-W1(3)*VT2(2)
      W1VT2(2)=-(W1(1)*VT2(3)-W1(3)*VT2(1))
      W1VT2(3)= W1(1)*VT2(2)-W1(2)*VT2(1)
C...
      W2VT1(1)= W2(2)*VT1(3)-W2(3)*VT1(2)
      W2VT1(2)=-(W2(1)*VT1(3)-W2(3)*VT1(1))
      W2VT1(3)= W2(1)*VT1(2)-W2(2)*VT1(1)
C...
      WV12(1)= W12(2)*V12(3)-W12(3)*V12(2)
      WV12(2)=-(W12(1)*V12(3)-W12(3)*V12(1))
      WV12(3)= W12(1)*V12(2)-W12(2)*V12(1)
```

```
C...
      V12S=0.0
      V12Q=0.0
      WNES=0.0
      WNEQ=0.0
      VWN= 0.0
      W1TN=0.0
      W2TN=0.0
      VT2N=0.0
C...
      DO 1 I=1,3
      V12S= V12(I)*ES(I)+V12S
      V12Q= V12(I)*EQ(I)+V12Q
      WNES= W12(I)*ESN(I)+WNES
      WNEQ= W12(I)*EQN(I)+WNEQ
      VWN = CN(I)*WV12(I)+VWN
      W1TN= CN(I)*W1VT2(I)+W1TN
      W2TN= CN(I)*W2VT1(I)+W2TN
      VT2N= CN(I)*VT2(I)+VT2N
  1   CONTINUE
C...
C...   COMPUTER THE CURVATURE OF THE GENERATED SURFACE
C...
      A13=-KS*V12S-WNES
      A23=-KQ*V12Q-WNEQ
      A33=KS*V12S**2+KQ*V12Q**2-VWN-W1TN+W2TN+PI21*VT2N/W2(3)
      T1=2.0D00*A13*A23
      T2=A23**2-A13**2-(KS-KQ)*A33
      SIG1F=0.5D00*DATAN2(T1,T2)
      KF=0.50D00*(KS+KQ)-0.5D00*(A13**2-A23**2)/A33
     &+A13*A23/(A33*DSIN(2.0D00*SIG1F))
      KH= KF-2.0D00*A13*A23/(A33*DSIN(2.0D00*SIG1F))
      SIGSF=SIG1F
      DO 2 I=1,3
      EF(I)= DCOS(SIG1F)*ES(I)-DSIN(SIG1F)*EQ(I)
      EH(I)= DSIN(SIG1F)*ES(I)+DCOS(SIG1F)*EQ(I)
  2   CONTINUE
      RETURN
      END
C
C...  FCN2 IS TO FIND THE INITIAL GEAR ROTATIONAL ANGLE
C
      SUBROUTINE FCN2(X,F,N)
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER N
      REAL*8 X(N),F(N)
      COMMON/A1/CNST,TN1,TN2,C,FW,GAMMA,XL,RL,MCD
      COMMON/A5/SG,XM,YM,ZM,XNM,YNM,ZNM,X2M,Y2M,Z2M,XN2M,YN2M,ZN2M,
     &XNH2,YNH2,ZNH2,XH2,YH2,ZH2
      CM=DCOS(GAMMA)
      SM=DSIN(GAMMA)
      CHP=DCOS(X(1))
      SHP=DSIN(X(1))
      XX= X2M
      YY=-Y2M*CHP+Z2M*SHP
      ZZ=-Y2M*SHP-Z2M*CHP
      XH2= XX*CM+ZZ*SM
      YH2= YY+C
      ZH2=-XX*SM+ZZ*CM
C...
```

```
      XX= XN2M
      YY=-YN2M*CHP+ZN2M*SHP
      ZZ=-YN2M*SHP-ZN2M*CHP
      XNH2= XX*CM+ZZ*SM
      YNH2= YY
      ZNH2=-XX*SM+ZZ*CM
C...
      R12=TN1/TN2
      V12X=-(YH2-C)*SM*R12
      V12Y= XH2*SM*R12+(1.0+R12*CM)*ZH2
      V12Z=-YH2*(1.0+R12*CM)+C*CM*R12
      F(1)=XNH2*V12X+YNH2*V12Y+ZNH2*V12Z
      RETURN
      END
C...
C...  THE FOLLOWING IS THE TCA SUBROUTINE FOR CURVED BLADE
C...
      SUBROUTINE FCNR(X,F,N)
      IMPLICIT REAL*8(A-H,O-Z)
      real*8 x(N),f(N)
      DIMENSION CH(3),P(3),E1EF(3),ESN(3),EQN(3),W1VT2(3),WV12(3),
     $W2VT1(3),EFIH(3),EFIIH(3),RH(3),GNH(3),E2IH(3),E2IIH(3),
     &E1IH(3),E1IIH(3),EFI(3),EFII(3),E1I(3),E1II(3),GN(3),EFE1(3),
     &ERR(20),XP(20),YP(20)
      COMMON/A1/CNST,TN1,TN2,C,FW,GAMMA,x1,r1,mcd
      COMMON/A2/B1,RGMA1,FGMA1,PGMA1,D1R,D1F,ADD1,DED1
      COMMON/A3/B2,RGMA2,FGMA2,PGMA2,D2R,D2F,ADD2,DED2,WD,CC,D2P
      COMMON/A4/SR2,Q2,RC2,PW2,XB2,XG2,EM2,GaMA2,CR2,ALP2,PHI2,PHI2P
      COMMON/A5/SG,XM,YM,ZM,XNM,YNM,ZNM,X2M,Y2M,Z2M,XN2M,YN2M,ZN2M,
     &XNH2,YNH2,ZNH2,XH2,YH2,ZH2
      COMMON/A6/ES(3),EQ(3),CN(3),W1(3),W2(3),W12(3),VT1(3),VT2(3),
     $V12(3),KS,KQ,KF,KH,EF(3),EH(3),SIGSF,PI21
      COMMON/A7/SR1,Q1,Rcf,PW1,XB1,XG1,EM1,GaMA1,CR1,ALP1,PHI1,PHI1P
      COMMON/A8/Sf,XM1,YM1,ZM1,XNM1,YNM1,ZNM1,X1M,Y1M,Z1M,
     &XN1M,YN1M,ZN1M,XNH1,YNH1,ZNH1,XH1,YH1,ZH1
      COMMON/A9/PHI2P0,OX,OZ,XO,ZO,RHO,ALP,V,H,CRIT,PCRIT
      COMMON/A11/RAM,PSI1,C2,D6,E24,F120,CX6,DX24,EX120,RU1,DELT,RUP,
     $RA1,CPF,DPF,EPF,FPF
      TH=X(1)
      PH=X(2)
      SP=DSIN(ALP2)
      CP=DCOS(ALP2)
      SM=DSIN(GAMA2)
      CM=DCOS(GAMA2)
      STP=DSIN(TH-PH)
      CTP=DCOS(TH-PH)
      XNM=-CP*CTP
      YNM=-CP*STP
      ZNM= SP
      AA1=RC2*STP+SR2*DSIN(-Q2-PH)
      AA2=RC2*CTP+SR2*DCOS(-Q2-PH)
      AX=-EM2*SM
      AY= XB2*CM
      AZ= EM2*CM
C
C...  FIND SG
C
      T1= XNM*(AX-AA1*(SM-CR2))+YNM*(AY+AA2*(SM-CR2))+ZNM*(AZ+AA1*CM)
      T2=-XNM*(SM-CR2)*SP*STP+YNM*((SM-CR2)*SP*CTP-CP*CM)+ZNM*CM*SP*STP
      SG=T1/T2
```

```
      XM= (RC2-SG*SP)*CTP+SR2*DCOS(-Q2-PH)
      YM= (RC2-SG*SP)*STP+SR2*DSIN(-Q2-PH)
      ZM=-SG*CP
C     XM=-SG*SP*CTP+AA2
C     YM=-SG*SP*STP+AA1
C     ZM=-SG*CP
      xX= CM*XM+SM*ZM-XG2-XB2*SM
      yY= YM+EM2
      zZ=-SM*XM+CM*ZM-XB2*CM
      XN= CM*XNM+SM*ZNM
      YN= YNM
      ZN=-SM*XNM+CM*ZNM
      PHI2=PH/CR2
      sh2=dsin(phi2)
      ch2=dcos(phi2)
      X2M= xX
      Y2M= CH2*yY-SH2*zZ
      Z2M= SH2*yY+CH2*zZ
      XN2M= XN
      YN2M= CH2*YN-SH2*ZN
      ZN2M= SH2*YN+CH2*ZN
      CMM=DCOS(GAMMA)
      SMM=DSIN(GAMMA)
      CHP=DCOS(PHI2P)
      SHP=DSIN(PHI2P)
      XX= X2M
      YY=-Y2M*CHP+Z2M*SHP
      ZZ=-Y2M*SHP-Z2M*CHP
      XH2= XX*CMM+ZZ*SMM
      YH2= YY+C+V
      ZH2=-XX*SMM+ZZ*CMM
C...
      XX= XN2M
      YY=-YN2M*CHP+ZN2M*SHP
      ZZ=-YN2M*SHP-ZN2M*CHP
      XNH2= XX*CMM+ZZ*SMM
      YNH2= YY
      ZNH2=-XX*SMM+ZZ*CMM

C...  DEFINE THE PINION SURFACE
C
      TH1=X(3)
      PH1=X(4)
      SM1=DSIN(GAMA1)
      CM1=DCOS(GAMA1)
      STP=DSIN(TH1+PH1)
      CTP=DCOS(TH1+PH1)
C
C...  FIND CRIT,PF,PPF,PCRIT
C
      DDD=DABS(PH1)
      IF(DDD.LE.0.001) GOTO 6
      PHI1=RA1*(PH1-CPF*PH1**2-DPF*PH1**3-EPF*PH1**4-FPF*PH1**5)
      PF=RA1*(1.0-2.0*CPF*PH1-3.0*DPF*PH1**2
     $-4.0*EPF*PH1**3-5.0*FPF*PH1**4)
      PPF=-RA1*(2.0*CPF+6.0*DPF*PH1+12.0*EPF*PH1**2+20.0*FPF*PH1**3)
      CRIT=1.0/PF
      PCRIT=-PPF/PF**3
      GOTO 7
 6    PHI1=RA1*PH1
```

```
      CR1T=CR1
      PCR1T=2.0*CPF/(RA1**2)
 7    CONTINUE
C     CR1T=CR1
C     PCR1T=0.000
C
C     FIND THE NOMAL OF THE EQUIDISTANCE SURFACE
C
      XMO= XO*CTP+SR1*DCOS(-Q1+PH1)
      YMO= XO*STP+SR1*DSIN(-Q1+PH1)
      ZMO= ZO
      V1X=-YMO*SM1-EM1*SM1
      V1Y= XMO*SM1-(ZMO-XB1)*CM1
      V1Z= YMO*CM1+EM1*CM1
      V2X=-YMO*CR1T
      V2Y= XMO*CR1T
      V2Z= 0.0
      VX=V1X-V2X
      VY=V1Y-V2Y
      VZ=V1Z-V2Z
      TX=-CTP
      TY=-STP
      TZ=0.0
      FX= STP
      FY=-CTP
      FZ=0.0
      XNN= FY*VZ-FZ*VX
      YNN= FZ*VX-FX*VZ
      ZNN= FX*VY-FY*VX
      DDD=DSQRT(XNN**2+YNN**2-ZNN**2)
      XNM1=XNN/DDD
      YNM1=YNN/DDD
      ZNM1=ZNN/DDD
      DT=TX*XNM1+TY*YNM1+TZ*ZNM1
      IF(DT.GE.0.0) GOTO 10
      XNM1=-XNM1
      YNM1=-YNM1
      ZNM1=-ZNM1
10    CONTINUE
      XM1= XMO-RHO*XNM1
      YM1= YMO-RHO*YNM1
      ZM1= ZMO-RHO*ZNM1
      ALP=DARCOS(TX*XNM1+TY*YNM1+TZ*ZNM1)
      xX= CM1*XM1+SM1*ZM1-XG1-XB1*SM1
      yY= YM1+EM1
      zZ=-SM1*XM1+CM1*ZM1-XB1*CM1
      XN1=CM1*XNM1+SM1*ZNM1
      YN1=YNM1
      ZN1=-SM1*XNM1+CM1*ZNM1
C     PHI1=PH1/CR1
      sh1=dsin(phi1)
      ch1=dcos(phi1)
      X1M= xX
      Y1M= CH1*yY+SH1*zZ
      Z1M=-SH1*yY+CH1*zZ
      XN1M= XN1
      YN1M= CH1*YN1+SH1*ZN1
      ZN1M=-SH1*YN1+CH1*ZN1
      PHI1P=X(5)
      sh1P=dsin(phi1?)
```

```
      ch1P=dcos(phi1P)
      XH1= X1M+H
      YH1= CH1P*Y1M-SH1P*Z1M
      ZH1= SH1P*Y1M+CH1P*Z1M
      XNH1= XN1M
      YNH1= CH1P*YN1M-SH1P*ZN1M
      ZNH1= SH1P*YN1M+CH1P*ZN1M
      F(1)=XH2-XH1
      F(2)=YH2-YH1
      F(3)=ZH2-ZH1
      F(4)=XNH2-XNH1
      F(5)=ZNH2-ZNH1
      RETURN
      END
C...
C...  THE FOLLOWING IS THE SUBROUTINE FOR STRAIGHT BLADE
C...
      SUBROUTINE FCN(X,F,N)
      IMPLICIT REAL*8(A-H,O-Z)
      real*8 x(N),f(N)
      DIMENSION CH(3),P(3),E1EF(3),ESN(3),EQN(3),W1VT2(3),WV12(3),
     $W2VT1(3),EFIH(3),EFIIH(3),RM(3),GNH(3),E2IH(3),E2IIH(3),
     &E1IH(3),E1IIH(3),EFI(3),EFII(3),E1I(3),E1II(3),GN(3),EFE1(3),
     &ERR(20),XP(20),YP(20)
      COMMON/A1/CNST,TN1,TN2,C,FW,GAMMA,x1,r1,mcd
      COMMON/A2/B1,RGMA1,FGMA1,PGMA1,D1R,D1F,ADD1,DED1
      COMMON/A3/B2,RGMA2,FGMA2,PGMA2,D2R,D2F,ADD2,DED2,WD,CC,D2P
      COMMON/A4/SR2,Q2,RC2,PW2,XB2,XG2,EM2,GaMA2,CR2,ALP2,PHI2,PHI2P
      COMMON/A5/SG,XM,YM,ZM,XNM,YNM,ZNM,X2M,Y2M,Z2M,XN2M,YN2M,ZN2M,
     &XNH2,YNH2,ZNH2,XH2,YH2,ZH2
      COMMON/A6/ES(3),EQ(3),CN(3),W1(3),W2(3),W12(3),VT1(3),VT2(3),
     $V12(3),KS,KQ,KF,KH,EF(3),EH(3),SIGSF,PI21
      COMMON/A7/SR1,Q1,Rcf,PW1,XB1,XG1,EM1,GaMA1,CR1,ALP1,PHI1,PHI1P
      COMMON/A8/SF,XM1,YM1,ZM1,XNM1,YNM1,ZNM1,X1M,Y1M,Z1M,
     &XN1M,YN1M,ZN1M,XNH1,YNH1,ZNH1,XH1,YH1,ZH1
      COMMON/A9/PHI2P0,OX,OZ,XO,ZO,RHO,ALP,V,H,CR1T,PCR1T
      COMMON/A11/RAM,PSI1,C2,D6,E24,F120,CX6,DX24,EX120,RU1,DELT,RUP
     $RA1,CPF,DPF,EPF,FPF
      TH=X(1)
      PH=X(2)
      SP=DSIN(ALP2)
      CP=DCOS(ALP2)
      SM=DSIN(GAMA2)
      CM=DCOS(GAMA2)
      STP=DSIN(TH-PH)
      CTP=DCOS(TH-PH)
      XNM=-CP*CTP
      YNM=-CP*STP
      ZNM= SP
      AA1=RC2*STP+SR2*DSIN(-Q2-PH)
      AA2=RC2*CTP+SR2*DCOS(-Q2-PH)
      AX=-EM2*SM
      AY= XB2*CM
      AZ= EM2*CM
C
C...  FIND SG
C
      T1= XNM*(AX-AA1*(SM-CR2))+YNM*(AY+AA2*(SM-CR2))+ZNM*(AZ+AA1*CM)
      T2=-XNM*(SM-CR2)*SP*STP+YNM*((SM-CR2)*SP*CTP-CP*CM)+ZNM*CM*SP*STP
      SG=T1/T2
```

```
      XM= (RC2-SG*SP)*CTP+SR2*DCOS(-Q2-PH)
      YM= (RC2-SG*SP)*STP+SR2*DSIN(-Q2-PH)
      ZM=-SG*CP
C     XM=-SG*SP*CTP+AA2
C     YM=-SG*SP*STP+AA1
C     ZM=-SG*CP
      xX= CM*XM+SM*ZM-XG2-XB2*SM
      yY= YM+EM2
      zZ=-SM*XM+CM*ZM-XB2*CM
      XN= CM*XNM+SM*ZNM
      YN= YNM
      ZN=-SM*XNM+CM*ZNM
      PHI2=PH/CR2
      sh2=dsin(phi2)
      ch2=dcos(phi2)
      X2M= xX
      Y2M= CH2*yY-SH2*zZ
      Z2M= SH2*yY+CH2*zZ
      XN2M= XN
      YN2M= CH2*YN-SH2*ZN
      ZN2M= SH2*YN+CH2*ZN
      CMM=DCOS(GAMMA)
      SMM=DSIN(GAMMA)
      CHP=DCOS(PHI2P)
      SHP=DSIN(PHI2P)
      XX= X2M
      YY=-Y2M*CHP+Z2M*SHP
      ZZ=-Y2M*SHP-Z2M*CHP
      XH2= XX*CMM+ZZ*SMM
      YH2= YY+C-V
C     YH2= YY+C-V
      ZH2=-XX*SMM+ZZ*CMM
C ..
      XX= XN2M
      YY=-YN2M*CHP+ZN2M*SHP
      ZZ=-YN2M*SHP-ZN2M*CHP
      XNH2= XX*CMM+ZZ*SMM
      YNH2= YY
      ZNH2=-XX*SMM+ZZ*CMM
C
C...  DEFINE THE PINION SURFACE
C
      TH1=X(3)
      PH1=X(4)
      SP1=DSIN(-ALP1)
      CP1=DCOS(-ALP1)
      SM1=DSIN(GAMA1)
      CM1=DCOS(GAMA1)
      STP=DSIN(TH1+PH1)
      CTP=DCOS(TH1+PH1)
      XNM1=-CP1*CTP
      YNM1=-CP1*STP
      ZNM1= SP1
      AB1=RCF*STP+SR1*DSIN(-Q1+PH1)
      AB2=RCF*CTP+SR1*DCOS(-Q1+PH1)
      AXX=-EM1*SM1
      AYY= XB1*CM1
      AZZ= EM1*CM1
C
C...  FIND SF,CRIT,PF,PPF,PCRIT
```

```
C
      DDD=DABS(PH1)
      IF(DDD.LE.0.001) GOTO 6
      PHI1=RA1*(PH1-CPF*PH1**2-DPF*PH1**3-EPF*PH1**4-FPF*PH1**5)
      PF=RA1*(1.0-2.0*CPF*PH1-3.0*DPF*PH1**2
     $-4.0*EPF*PH1**3-5.0*FPF*PH1**4)
      PPF=-RA1*(2.0*CPF+6.0*DPF*PH1+12.0*EPF*PH1**2+20.0*FPF*PH1**3)
      CRIT=1.0/PF
      PCRIT=-PPF/PF**3
      GOTO 7
 6    PHI1=RA1*PH1
      CRIT=CR1
      PCRIT=2.0*CPF/(RA1**2)
 7    CONTINUE
C     CRIT=CR1
C     PCRIT=0.000
      T1= XNM1*(AXX-AB1*(SM1-CRIT))+
     &YNM1*(AYY+AB2*(SM1-CRIT))-ZNM1*(AZZ-AB1*CM1)
      T2=-XNM1*(SM1-CRIT)*SP1*STP+
     &YNM1*((SM1-CRIT)*SP1*CTP-CP1*CM1)-ZNM1*CM1*SP1*STP
      SF=T1/T2
C
      XM1= (RCF-SF*SP1)*CTP+SR1*DCOS(-Q1+PH1)
      YM1= (RCF-SF*SP1)*STP-SR1*DSIN(-Q1+PH1)
      ZM1=-SF*CP1
      xX= CM1*XM1+SM1*ZM1-XG1-XB1*SM1
      yY= YM1-EM1
      zZ=-SM1*XM1+CM1*ZM1-XB1*CM1
      XN1=CM1*XNM1+SM1*ZNM1
      YN1=YNM1
      ZN1=-SM1*XNM1+CM1*ZNM1
C     PHI1=PH1/CR1
      sh1=dsin(phi1)
      ch1=dcos(phi1)
      X1M= xX
      Y1M= CH1*yY+SH1*zZ
      Z1M=-SH1*yY+CH1*zZ
      XN1M= XN1
      YN1M= CH1*YN1-SH1*ZN1
      ZN1M=-SH1*YN1+CH1*ZN1
C     WRITE(9,111) X1M,Y1M,Z1M
      PHI1P= X(5)
      sh1P=dsin(phi1P)
      ch1P=dcos(phi1P)
      XH1= X1M+H
      YH1= CH1P*Y1M-SH1P*Z1M
      ZH1= SH1P*Y1M+CH1P*Z1M
      XNH1= XN1M
      YNH1= CH1P*YN1M-SH1P*ZN1M
      ZNH1= SH1P*YN1M+CH1P*ZN1M
      F(1)=XH2-XH1
      F(2)=YH2-YH1
      F(3)=ZH2-ZH1
      F(4)=XNH2-XNH1
      F(5)=YNH2-YNH1
      RETURN
      END
C...                                                                    AA 00010
C...  ***************************************************************   AA 00020
C...  * SUBROUTINE ELLIP IS TO DETERMINE THE SIZE AND ORINTATION O      AA 00030
```

```
C...  *                 OF THE CONTACT ELLIPSE                          *
C...  ***************************************************************
C...
      SUBROUTINE ELLIP
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 KS,KQ,K1I,K1II,K2I,K2II
      DIMENSION RO(3),ETA2(3),ZETA2(3),E1E2(3),ETA(3),ZETA(3)
      DIMENSION E1IH(3),E1IIH(3),E2IH(3),E2IIH(3),GNH(3)
      COMMON/A1/CNST,TN1,TN2,C,FW,GAMMA,x1,r1,mcd
      COMMON/A3/B2,RGMA2,FGMA2,PGMA2,D2R,D2F,ADD2,DED2,WD,CC,D2P
      COMMON/A4/SR2,Q2,RC2,PW2,XB2,XG2,EM2,GaMA2,CR2,ALP2,PHI2,PHI2P
      COMMON/A5/SG,XM,YM,ZM,XNM,YNM,ZNM,X2M,Y2M,Z2M,XN2M,YN2M,ZN2M,
     &XNH2,YNH2,ZNH2,XH2,YH2,ZH2
      COMMON/A9/PHI2P0,OX,OZ,XO,ZO,RHO,ALP,V,H,CRIT,PCRIT
      COMMON/A10/K1I,K1II,K2I,K2II,DEL,E1IH,E1IIH,E2IH,E2IIH,GNH,
     &A2P,B2P,TAU1R,TAU2R,A2L,B2L
      CNST=DARCOS(-1.0D00)/180.00
C...
      E1E2(1)= E1IH(2) *E2IH(3)-E1IH(3)*E2IH(2)
      E1E2(2)=-(E1IH(1)*E2IH(3)-E1IH(3)*E2IH(1))
      E1E2(3)= E1IH(1) *E2IH(2)-E1IH(2)*E2IH(1)
C...
      T1=0.0
      T2=0.0
      DO 1 I=1,3
      T1= E1IH(I)*E2IH(I)+T1
      T2= GNH(I)*E1E2(I)+T2
    1 CONTINUE
C...
      T1=T1+1.0D00
      SIG12=2.0D00*DATAN2(T2,T1)
C...
      SK1= K1I+K1II
      SK2= K2I+K2II
      SG1= K1I-K1II
      SG2= K2I-K2II
C...
      T1=SG1-SG2*DCOS(2.0D00*SIG12)
      T2=SG2*DSIN(2.0D00*SIG12)
      T3=DSQRT(SG1**2+SG2**2-2.0D00*SG1*SG2*DCOS(2.0D00*SIG12))
C...
      TX=T2/T3
      TY=T1/T3+1.0D00
      ALP12=DATAN2(TX,TY)
C...
C...  THE DIRECTION AND LENGTH OF THE AXES OF CONTACT ELLIPSE
C...
C     DEL=0.00700D00
      AL=0.25D00*(SK1-SK2-T3)
      BL=0.25D00*(SK1-SK2+T3)
C     WRITE(9,5) SIG12,AL,BL
C 5   FORMAT(2X,'SIG12, AL,BL = ',3(2X,G14.7))
      AL=DABS(AL)
      BL=DABS(BL)
      A2L=2.0D00*DSQRT(DEL/AL)
      B2L=2.0D00*DSQRT(DEL/BL)
C...
C...
      DO 2 I=1,3
      ETA(I) = DCOS(ALP12)*E1IH(I)-DSIN(ALP12)*E1IIH(I)
```

```
      ZETA(I)= DSIN(ALP12)*E1IH(I)+DCOS(ALP12)*E1IIH(I)
    2 CONTINUE
C...
C...  DETERMINE THE PROJECTION OF CONTACT ELLIPS IN AXIAL SECTION
C...
      CHP=DCOS(PHI2P)
       SHP=DSIN(PHI2P)
C...
      CMM=DCOS(GAMMA)
      SMM=DSIN(GAMMA)
C...
      XX= ETA(1)*CMM-ETA(3)*SMM
      YY= ETA(2)
      ZZ= ETA(1)*SMM+ETA(3)*CMM
      ETA2(1)= XX
      ETA2(2)=-YY*CHP-ZZ*SHP
      ETA2(3)= YY*SHP-ZZ*CHP
C...
      XX= ZETA(1)*CMM-ZETA(3)*SMM
      YY= ZETA(2)
      ZZ= ZETA(1)*SMM+ZETA(3)*CMM
      ZETA2(1)= XX
      ZETA2(2)=-YY*CHP-ZZ*SHP
      ZETA2(3)= YY*SHP-ZZ*CHP
C...
      RO(2)=Y2M/DSQRT(Z2M**2+Y2M**2)
      RO(3)=Z2M/DSQRT(Z2M**2+Y2M**2)
      RO(1)=0.0D00
C...
      T11=0.0D00
       T12=0.0D00
      DO 3 I=1,3
      T12= ETA2(I)*RO(I)+T12
      T11=ZETA2(I)*RO(I)+T11
    3 CONTINUE
C...
      TAU1=DATAN2(T11,ZETA2(1))
      TAU2=DATAN2(T12,ETA2(1))
C...
      A2P=A2L*ZETA2(1)/DCOS(TAU1)
      B2P=B2L*ETA2(1)/DCOS(TAU2)
C...
      TAU1R=(TAU1-RGMA2)/CNST
      TAU2R=(TAU2-RGMA2)/CNST
      RETURN
      END
C...
C...  THE FOLLOWING IS THE V-H CHECK PROGRAM FOR CURVED BLADE
C...
      SUBROUTINE FCNMR(X,F,N)
      IMPLICIT REAL*8(A-H,O-Z)
      real*8 x(N),f(N)
      COMMON/A1/CNST,TN1,TN2,C,FW,GAMMA,x1,r1,mcd
      COMMON/A5/SG,XM,YM,ZM,XNM,YNM,ZNM,X2M,Y2M,Z2M,XN2M,YN2M,ZN2M,
     &XNH2,YNH2,ZNH2,XH2,YH2,ZH2
      COMMON/A7/SR1,Q1,Rcf,PW1,XB1,XG1,EM1,GaMA1,CR1,ALP1,PHI1,PHI1P
      COMMON/A9/PHI2P0,OX,OZ,XO,ZO,RHO,ALP,V,H,CRIT,PCRIT
      COMMON/A11/RAM,PSI1,C2,D6,E24,F120,CX6,DX24,EX120,RU1,DELT,RUP,
     $RA1,CPF,DPF,EPF,FPF
      CM=DCOS(GAMMA)
```

```
      SM=DSIN(GAMMA)
C     CHP=DCOS(PHI2P0)
C     SHP=DSIN(PHI2P0)
      CHP=DCOS(X(1))
      SHP=DSIN(X(1))
      XX= X2M
      YY=-Y2M*CHP+Z2M*SHP
      ZZ=-Y2M*SHP-Z2M*CHP
      XH2= XX*CM+ZZ*SM
      YH2= YY+C
      ZH2=-XX*SM+ZZ*CM
C...
      XX= XN2M
      YY=-YN2M*CHP+ZN2M*SHP
      ZZ=-YN2M*SHP-ZN2M*CHP
      XNH2= XX*CM+ZZ*SM
      YNH2= YY
      ZNH2=-XX*SM+ZZ*CM

C...  DEFINE THE PINION SURFACE
C
      TH1=X(2)
      PH1=X(3)
      SM1=DSIN(GAMA1)
      CM1=DCOS(GAMA1)
      STP=DSIN(TH1+PH1)
      CTP=DCOS(TH1+PH1)
C
C...  FIND CRIT,PF,PPF,PCRIT
C
      DDD=DABS(PH1)
      IF(DDD.LE.0.001) GOTO 6
      PHI1=RA1*(PH1-CPF*PH1**2-DPF*PH1**3-EPF*PH1**4-FPF*PH1**5)
      PF=RA1*(1.0-2.0*CPF*PH1-3.0*DPF*PH1**2
     $-4.0*EPF*PH1**3-5.0*FPF*PH1**4)
      PPF=-RA1*(2.0*CPF+6.0*DPF*PH1-12.0*EPF*PH1**2-20.0*FPF*PH1**3)
      CRIT=1.0/PF
      PCRIT=-PPF/PF**3
      GOTO 7
 6    PHI1=RA1*PH1
      CRIT=CR1
      PCRIT=2.0*CPF/(RA1**2)
 7    CONTINUE
C
C     FIND THE NOMAL OF THE EQUIDISTANCE SURFACE
C
      XMO= XO*CTP+SR1*DCOS(-Q1+PH1)
      YMO= XO*STP+SR1*DSIN(-Q1+PH1)
      ZMO= ZO
      V1X=-YMO*SM1-EM1*SM1
      V1Y= XMO*SM1-(ZMO-XB1)*CM1
      V1Z= YMO*CM1+EM1*CM1
      V2X=-YMO*CRIT
      V2Y= XMO*CRIT
      V2Z= 0.0
      VX=V1X-V2X
      VY=V1Y-V2Y
      VZ=V1Z-V2Z
      TX=-CTP
      TY=-STP
```

```
      TZ=0.0
      FX= STP
      FY=-CTP
      FZ=0.0
      XNN= FY*VZ-FZ*VX
      YNN= FZ*VX-FX*VZ
      ZNN= FX*VY-FY*VX
      DDD=DSQRT(XNN**2+YNN**2+ZNN**2)
      XNM1=XNN/DDD
      YNM1=YNN/DDD
      ZNM1=ZNN/DDD
      DT=TX*XNM1+TY*YNM1+TZ*ZNM1
      IF(DT.GE.0.0) GOTO 10
      XNM1=-XNM1
      YNM1=-YNM1
      ZNM1=-ZNM1
 10   CONTINUE
      XM1= XMO-RHO*XNM1
      YM1= YMO-RHO*YNM1
      ZM1= ZMO-RHO*ZNM1
      ALP=DARCOS(TX*XNM1+TY*YNM1+TZ*ZNM1)
      xX= CM1*XM1+SM1*ZM1-XG1-XB1*SM1
      yY= YM1+EM1
      zZ=-SM1*XM1+CM1*ZM1-XB1*CM1
      XN1=CM1*XNM1+SM1*ZNM1
      YN1=YNM1
      ZN1=-SM1*XNM1+CM1*ZNM1
C     PHI1=PH1/CR1
      sh1=dsin(phi1)
      ch1=dcos(phi1)
      X1M= xX
      Y1M= CH1*yY-SH1*zZ
      Z1M=-SH1*yY+CH1*zZ
      XN1M= XN1
      YN1M= CH1*YN1+SH1*ZN1
      ZN1M=-SH1*YN1+CH1*ZN1
C...
C
      TT=YN1M**2+ZN1M**2
      SH1P=(-ZN1M*YNH2+YN1M*ZNH2)/TT
      CH1P=( YN1M*YNH2+ZN1M*ZNH2)/TT
      PHI1P=2.0D00*DATAN2(SH1P,(1.0D00+CH1P))
C...
C...
      XH1= X1M
      YH1= CH1P*Y1M-SH1P*Z1M
      ZH1= SH1P*Y1M+CH1P*Z1M
      XNH1= XN1M
      YNH1= CH1P*YN1M-SH1P*ZN1M
      ZNH1= SH1P*YN1M+CH1P*ZN1M
      V=-(YH2-YH1)
      H=XH2-XH1
C...
      F(1)=ZH2-ZH1
      F(2)=XNH2-XNH1
C     F(2)=YNH2**2+ZNH2**2-TT
C
C
C...
      R12=TN1/TN2
```

```
      V12X=0.0-YH2*SM*R12
      V12Y= ZH1+R12*(XH2*SM+ZH2*CM)
      V12Z=-YH1-R12*YH2*CM
C     V12X=-(YH2-(C-V))*SM*R12
C     V12Y= XH2*SM*R12+(1.0+R12*CM)*ZH2
C     V12Z=-YH2*(1.0+R12*CM)+(C-V)*CM*R12
      F(3)=XNH2*V12X+YNH2*V12Y+ZNH2*V12Z
      RETURN
      END
C...
C...  THE FOLLOWING IS THE V-H CHECK SUBROUTINE FOR STRAIGHT BLADE
C...
      SUBROUTINE FCNM(X,F,N)
      IMPLICIT REAL*8(A-H,O-Z)
      real*8 x(N),f(N)
      COMMON/A1/CNST,TN1,TN2,C,FW,GAMMA,x1,r1,mcd
      COMMON/A5/SG,XM,YM,ZM,XNM,YNM,ZNM,X2M,Y2M,Z2M,XN2M,YN2M,ZN2M,
     &XNH2,YNH2,ZNH2,XH2,YH2,ZH2
      COMMON/A7/SR1,Q1,Rcf,PW1,XB1,XG1,EM1,GaMA1,CR1,ALP1,PHI1,PHI1P
      COMMON/A9/PHI2P0,OX,OZ,XO,ZO,RHO,ALP,V,H,CR1T,PCR1T
      COMMON/A11/RAM,PSI1,C2,D6,E24,F120,CX6,DX24,EX120,RU1,DELT,RUP,
     $RA1,CPF,DPF,EPF,FPF
      CM=DCOS(GAMMA)
      SM=DSIN(GAMMA)
      CHP=DCOS(X(1))
      SHP=DSIN(X(1))
      XX= X2M
      YY=-Y2M*CHP-Z2M*SHP
      ZZ=-Y2M*SHP-Z2M*CHP
      XH2= XX*CM+ZZ*SM
      YH2= YY+C
      ZH2=-XX*SM-ZZ*CM
C...
      XX= XN2M
      YY=-YN2M*CHP+ZN2M*SHP
      ZZ=-YN2M*SHP-ZN2M*CHP
      XNH2= XX*CM+ZZ*SM
      YNH2= YY
      ZNH2=-XX*SM+ZZ*CM
C...
C
C...  DEFINE THE PINION SURFACE
C
      TH1=X(2)
      PH1=X(3)
      SP1=DSIN(-ALP1)
      CP1=DCOS(-ALP1)
      SM1=DSIN(GAMA1)
      CM1=DCOS(GAMA1)
      STP=DSIN(TH1+PH1)
      CTP=DCOS(TH1+PH1)
      XNM1=-CP1*CTP
      YNM1=-CP1*STP
      ZNM1= SP1
      AB1=RCF*STP+SR1*DSIN(-Q1+PH1)
      AB2=RCF*CTP+SR1*DCOS(-Q1+PH1)
      AXX=-EM1*SM1
      AYY= XB1*CM1
      AZZ= EM1*CM1
C
```

```
C...  FIND SF,CRIT,PF,PPF,PCRIT
C
      PHII=RA1*(PH1-CPF*PH1**2-DPF*PH1**3-EPF*PH1**4-FPF*PH1**5)
      PF=RA1*(1.0-2.0*CPF*PH1-3.0*DPF*PH1**2
     $-4.0*EPF*PH1**3-5.0*FPF*PH1**4)
      PPF=-RA1*(2.0*CPF+6.0*DPF*PH1+12.0*EPF*PH1**2+20.0*FPF*PH1**3)
      CRIT=1.0/PF
      PCRIT=-PPF/PF**3
C
      T1= XNM1*(AXX-AB1*(SM1-CRIT))+
     &YNM1*(AYY+AB2*(SM1-CRIT))+ZNM1*(AZZ+AB1*CM1)
      T2=-XNM1*(SM1-CRIT)*SP1*STP+
     &YNM1*((SM1-CRIT)*SP1*CTP-CP1*CM1)+ZNM1*CM1*SP1*STP
      SF=T1/T2
      XM1= (RCF-SF*SP1)*CTP+SR1*DCOS(-Q1+PH1)
      YM1= (RCF-SF*SP1)*STP+SR1*DSIN(-Q1+PH1)
      ZM1=-SF*CP1
      xX= CM1*XM1+SM1*ZM1-XG1-XB1*SM1
      yY= YM1+EM1
      zZ=-SM1*XM1+CM1*ZM1-XB1*CM1
      XN1=CM1*XNM1+SM1*ZNM1
      YN1=YNM1
      ZN1=-SM1*XNM1+CM1*ZNM1
C     PHI1=PH1/CR1
      sh1=dsin(phi1)
      ch1=dcos(phi1)
      X1M= xX
      Y1M= CH1*yY+SH1*zZ
      Z1M=-SH1*yY-CH1*zZ
      XN1M= XN1
      YN1M= CH1*YN1+SH1*ZN1
      ZN1M=-SH1*YN1+CH1*ZN1
      TT=YN1M**2+ZN1M**2
      SH1P=(-ZN1M*YNH2+YN1M*ZNH2)/TT
      CH1P=( YN1M*YNH2+ZN1M*ZNH2)/TT
      PHI1P=2.0D00*DATAN2(SH1P,(1.0D00+CH1P))
C     SH1P=DSIN(PHI1P)
C     CH1P=DCOS(PHI1P)
      XH1= X1M
      YH1= CH1P*Y1M-SH1P*Z1M
      ZH1= SH1P*Y1M+CH1P*Z1M
      XNH1= XN1M
      YNH1= CH1P*YN1M-SH1P*ZN1M
      ZNH1= SH1P*YN1M+CH1P*ZN1M
      V=-(YH2-YH1)
      H=XH2-XH1
C...
      F(1)=ZH2-ZH1
      F(2)=XNH2-XNH1
C     F(2)=YNH2**2+ZNH2**2-TT
C
C
C...
      R12=TN1/TN2
      V12X=0.0-YH2*SM*R12
      V12Y= ZH1+R12*(XH2*SM+ZH2*CM)
      V12Z=-YH1-R12*YH2*CM
C     V12X=-(YH2-(C-V))*SM*R12
C     V12Y= XH2*SM*R12+(1.0+R12*CM)*ZH2
C     V12Z=-YH2*(1.0+R12*CM)+(C-V)*CM*R12
```

```
      F(3)=XNH2*V12X+YNH2*V12Y+ZNH2*V12Z
      RETURN
      END
C
C     SUBROUTINE CAM IS FOR THE COEFFICIENTS OF GENERATION MOTION
C
      SUBROUTINE CAM                                                    AA 00070
      IMPLICIT REAL*8(A-H,O-Z)                                          AA 00080
      COMMON/A11/RAM,PSI1,C2,D6,E24,F120,CX6,DX24,EX120,RU1,DELT,RUP,
     $RA1,CPF,DPF,EPF,FPF
      T1=1.0+3.0*C2*DTAN(PSI1)
     &+(1.0-RAM)**3*(RU1**3/15.0**2-DELT)/(15.0*DCOS(PSI1))
      T2=1.0+(RU1+DELT)/(15.0*DCOS(PSI1))
      CX6=T1/T2
      T1= 6.0*C2*DCOS(PSI1)+(4.0*CX6+3.0*C2**2-1.0)*DSIN(PSI1)
     &-6.0*C2*(1.0-RAM)**2*(RU1**3/15.0**3+DELT/15.0)
      T2= DCOS(PSI1)+(RU1+DELT)/15.0
      DX24= T1/T2
      T1=(10.0*CX6+15.0*C2**2-1.0)*DCOS(PSI1)
     &  -(5.0*DX24+10.0*C2*CX6-10.0*C2)*DSIN(PSI1)
     &  +(10.0*CX6*(1.0-RAM)**2
     &   -15.0*C2**2*(1.0-RAM))*(RU1**3/15.0**3+DELT/15.0)
     &  -(1.0-RAM)**5*(RU1**5/15.0**5+DELT/15.0)
      T2=DCOS(PSI1)+(RU1+DELT)/15.0
      EX120=T1/T2
      D6=CX6-3.0*C2**2
      E24=DX24+C2*(15.0*C2**2-10.0*CX6)
      F120=EX120-15.0*C2*DX24-105.0*C2**2*(CX6-C2**2 -10.0*CX6**2
      CPF=C2/2.0
      DPF=D6/6.0
      EPF=E24/24.0
      FPF=F120/120.0
      RETURN
      END
```

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16. Abstract

Computerized simulation of meshing and bearing contact for spiral bevel gears and hypoid gears is a significant achievement that could improve substantially the technology and the quality of the gears. This report covers a new approach to the synthesis of face-milled spiral bevel gears and their tooth contact analysis. The proposed approach is based on the following ideas: (i) application of the principle of local synthesis that provides optimal conditions of meshing and contact at the mean contact point M and in the neighborhood of M; (ii) application of relations between principle directions and curvatures for surfaces being in line contact or in point contact. The developed local synthesis of gears provides (i) the required gear ratio at M; (ii) a localized bearing contact with the desired direction of the tangent to the contact path on gear tooth surface and the desired length of the major axis of contact ellipse at M; (iii) a predesigned parabolic function of a controlled level (8-10 arc seconds) for transmission errors; such a function of transmission errors enables to absorb linear functions of transmission errors caused by misalignment and reduce the level of vibrations. The proposed approach does not require either the tilt of the head-cutter for the process of generation or modified roll for the pinion generation. Improved conditions of meshing and contact of the gears can be achieved without the above mentioned parameters. The report is complemented with a computer program for determination of basic machine-tool settings and tooth contact analysis for the designed gears. The approach is illustrated with a numerical example.

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